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Applying the CRADIS Method with Probabilistic Uncertain Linguistic T-Spherical Fuzzy Sets for Circular Supplier Selection

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ABSTRACT

This paper proposes a multi-criteria decision-making model based on the Probabilistic Uncertain Linguistic T-Spherical Fuzzy Set (PULTSFS), integrating FUCOM, ITARA, and CRADIS, to address the ambiguity and uncertainty inherent in evaluation information for circular supplier selection. It also tackles the limitations of existing CRADIS methods in complex fuzzy environments, the lack of a scientific mechanism that synergizes subjective and objective weighting, and the incomplete depiction of evaluation information. First, a Euclidean distance measure is defined based on the relevant concepts of PULTSFS. Next, the subjective weights of the criteria are derived using the Full Consistency Method (FUCOM), while the objective weights are determined by extending the ITARA method within the PULTSFS framework, thereby establishing a combined weighting mechanism that integrates subjective and objective considerations. Finally, a multi-criteria decision-making model is constructed based on the novel PULTSF-CRADIS approach. Using circular supplier selection as a case study, sensitivity analysis and comparative validation demonstrate that the proposed model yields robust and reliable ranking results, with significantly enhanced capability in handling uncertain information and improved adaptability to complex decision-making scenarios, offering a more precise decision-making tool for supplier selection in the context of the circular economy.

1. Introduction

Global commitments to climate change mitigation are deepening. Sustainable transport and green energy transition have become core priorities for national development. Against this backdrop, rapid growth in the new energy sector has optimized energy structures. It has also created severe challenges for resource security in underlying supply chains. Circular supplier selection serves as a critical decision for achieving closed-loop resource management[1]. It directly affects corporate environmental performance[2], resource efficiency[3], and long-term competitiveness[4]. However, this decision process is highly complex. It typically involves multiple conflicting economic, environmental, and social criteria. Decision-makers often face cognitive vagueness, incomplete information, and preference expressions characterized by hesitation and neutrality. Therefore, accurately characterizing the fuzziness and hesitation in evaluation information to rationally rank circular suppliers represents an urgent challenge for both management practice and academic research.

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Circular supplier selection (CSS) is essentially a complex multi-criteria decision-making (MCDM) problem. To address this challenge, scholars have extended classical fuzzy sets[5], intuitionistic fuzzy sets[6], Pythagorean fuzzy sets[7], and Fermatean fuzzy sets [8]. These approaches use two dimensions—membership and non-membership degrees—to characterize evaluation information. Additionally, q-rung orthopair fuzzy sets (q-ROFS)[9] provide a more generalized concept. They relax the constraint to the qth power sum of membership and non-membership degrees being less than or equal to 1. This enables broader fuzzy information representation with strong operability. However, in certain decision scenarios, people may express opinions beyond support and opposition. They may also include some degree of "neutral" information. To handle such cases, scholars have proposed three-dimensional forms involving membership, neutrality, and non-membership degrees. These fully and accurately characterize the fuzziness and uncertainty in evaluation information. Examples include picture fuzzy sets[10], spherical fuzzy sets[11], and T-spherical fuzzy sets (T-SFS) [12] T-spherical fuzzy sets satisfy the condition that the qth power sum of membership, neutrality, and non-membership degrees is less than or equal to 1 ($q \geqslant 1$). Compared with picture fuzzy sets and spherical fuzzy sets, T-spherical fuzzy sets allow experts to express views on evaluated objects more comprehensively and freely. This provides greater flexibility for experts. Consequently, T-spherical fuzzy sets serve as a powerful tool for effectively characterizing and processing information fuzziness.

Existing extensions of fuzzy set theory primarily address uncertain decision problems from quantitative perspectives. However, when facing complex uncertain decision problems, people often struggle to express their views through precise numerical values based solely on experience and intuition. Qualitative linguistic evaluations such as "promising prospects," "high expected returns," or "substantial risks" align better with human cognition. To address this, Zadeh[13],[14],[15] pioneered the linguistic term set (LTS), laying the foundation for qualitative assessment. Subsequently, Herrera[16] and Martinez proposed the 2-tuple linguistic model. This model effectively overcomes information loss during aggregation through a symbolic translation mechanism. However, as decision problems grow increasingly complex, single terms fail to capture the "hesitation" experts experience when weighing multiple alternatives. To tackle this, Rodriguez et al.[17] introduced hesitant fuzzy linguistic term sets (HFLTS). This approach allows decision-makers to express preferences as sets, achieving a shift from single-value to multi-value representation. In recent years, research focus has shifted toward refining the characterization of group assessment information distributions. Pang[18] innovatively proposed probabilistic linguistic term sets (PLTS). By assigning probability weights to terms, this significantly enhances the model's ability to quantify group consensus and divergence. Lin et al.[19]then integrated this framework with interval uncertainty, defining probabilistic uncertain linguistic term sets (PULTS). This enables comprehensive handling of fuzziness, hesitation, and statistical probability simultaneously. Although these models have achieved significant progress in information expression, existing PULTS and its derivatives—such as PULIFS proposed by Gong and Chen[20] and PULq-ROFS proposed by Naz et al. [21]—still face limitations when handling extremely complex evaluation scenarios. Most of these models rely on two-dimensional "membership-non-membership" architectures. They struggle to effectively represent "indeterminacy" (neutral degrees) in evaluation information or abstention preferences within hesitation. In many real-world decision scenarios, neutral information constitutes not only a vital component of expert attitudes but also a critical indicator for measuring decision risks. Few studies in existing literature have integrated PULTS with T-SFS. Publications co-authored by the authors of this paper[22][23] first proposed and preliminarily constructed the integrated concept of PULTSFS. This study builds upon that concept to deeply expand relevant research directions.

Additionally, at the methodological level of decision analysis, Pamučar et al.[24]proposed the Full Consistency Method (FUCOM) in 2018. This method offers a concise and efficient approach for determining subjective criterion weights in MCDM models. FUCOM emerged from researchers' reflections on the limitations of prevailing subjective weighting methods at that time. The Analytic Hierarchy Process (AHP)[25] struggles to ensure complete pairwise consistency when handling numerous criteria (e.g., exceeding nine). The Best-Worst Method (BWM) [26]reduces comparison frequency, yet its consistency ratio settings may still reflect high subjectivity. The DEMATEL[27] method analyzes relationships among criteria but lacks measures for result consistency. Against this backdrop, FUCOM resolves the strong subjectivity and cumbersome consistency testing of these methods by maximizing consistency degrees of criterion weights. Subsequently, it has been applied to frontier domains such as circular supplier selection. It provides scientific and reliable bases for weight allocation across multi-dimensional criteria in this context. In practical applications, scholars have integrated FUCOM with other MCDM methods to form hybrid models. For instance, A. R. Mishra [28]combined FUCOM with MABAC for manufacturer selection and ranking. AbdEllatif[29]integrated it with TOPSIS for supply chain supplier evaluation. Puška [30] combined it with COPRAS for healthcare waste incineration assessment. Hashemkhani[31]incorporated it into WASPAS for warehouse loading and unloading equipment evaluation.

Although FUCOM significantly reduces instability in subjective judgments during weight determination, it remains essentially a subjective weighting method. This means it struggles to fully reflect objective discrepancy information embedded in decision data. Consequently, researchers have increasingly favored combining subjective and objective weighting methods to enhance the scientific rigor of weight allocation. In 2019, Hatefi[32] proposed the Indifference Threshold-based Attribute Ratio Analysis (ITARA) method. He introduced the indifference threshold into weight calculation, constructing a semi-objective weighting approach that integrates data dispersion with threshold screening mechanisms. The core of ITARA involves analyzing adjacent differences after ranking alternatives and retaining only effective deviations exceeding the indifference threshold. This enables weights to better reflect the discriminating power of criteria[33]. Scholars subsequently extended ITARA to various domains, such as combining it with CCSD for logistics equipment selection [34] and integrating it with TOPSIS for risk assessment[33]. It demonstrates strong robustness across different contexts. Recently, ITARA innovations have shifted from application-level advancements to methodological refinements. On one hand, researchers have extended it to fuzzy environments, including intuitionistic and Fermatean fuzzy settings[35]. On the other hand, improved versions like M-ITARA[36]have been proposed to address traditional ITARA's neglect of ideal solution information and criterion interdependence. These enhancements introduce ideal levels and correlation structures to strengthen global discriminating capability. Nevertheless, persistent issues remain unresolved, including the subjective setting of indifference thresholds and the limitation of deviation measures to local ranking differences. In classical MCDM methods, Euclidean distance has proven effective in characterizing the proximity between alternatives and ideal solutions. It essentially reflects the overall discrepancy structure in multi-dimensional attribute spaces. Therefore, introducing Euclidean distance into the ITARA framework promises to overcome its limitation of being driven solely by local differences.

Meanwhile, CRADIS emerged as a novel multi-attribute decision-making algorithm first proposed by Puška et al.[30] in 2021. It integrates the "optimal solution" concept from ARAS [37], deviation measurement methods from MARCOS[38] and TOPSIS[39], and utility functions. The method systematically calculates comprehensive deviations between alternatives and both ideal and anti-ideal solutions. It constructs a compromise ranking model based on distances to ideal solutions. This effectively avoids rank reversal issues prevalent in traditional MCDM methods. Since its inception, scholars have continuously extended CRADIS to fuzzy environments and applied it to increasingly complex scenarios. Subsequently, Raghunathan and Ecer[40]advanced CRADIS under q-ROFS environments to better handle fuzziness and uncertainty in human cognition. Since 2023, CRADIS applications have expanded significantly. It is frequently combined with mainstream weighting methods such as MEREC[41], LOPCOW [42], and CRITIC[43]. Existing studies demonstrate its strong applicability in multi-objective balanced decision scenarios, including food and industrial decisions [44][45]. However, systematic applications in circular supplier evaluation remain underexplored. In summary, integrating FUCOM's subjective approach with ITARA's objective weighting to determine criterion weights, and combining this with CRADIS's robust ranking logic, promises to provide a computationally efficient, reliable, and highly interpretable decision-making pathway for circular supplier selection.

This study addresses three unresolved practical challenges in the field. First, existing circular supplier selection research lacks sufficient information expression capability to characterize complex uncertainty. In complex group decision-making scenarios, experts possess significantly different knowledge backgrounds and preferences. Traditional fuzzy sets or single linguistic information expression methods struggle to capture these nuances accurately. This is particularly problematic when involving interval linguistic terms, where information distortion occurs easily. Therefore, introducing a more finely structured information representation is necessary. Second, criterion weight determination mechanisms remain inadequate, with insufficient research on integrating subjective and objective methods. Single subjective weighting methods are vulnerable to individual influences, while single objective methods fail to reflect actual decision semantics. Under MCDM contexts, studies lack weighting mechanisms that combine both advantages. Thus, constructing a method that integrates subjective and objective information is essential. Third, CRADIS ranking methods demonstrate insufficient scenario adaptability. Classical CRADIS fails to adequately consider highly uncertain information environments. It exhibits inadequate ranking robustness when facing complex fuzzy linguistic evaluations and cannot be directly applied to circular supplier decision scenarios under novel information expression frameworks such as PULTSFS. Consequently, expanding and modifying CRADIS to enhance its adaptability to new uncertain expression frameworks is imperative.

To address these unresolved challenges, this study focuses on practical decision-making needs in circular supplier selection. It aims to propose targeted improvement schemes. The research contributions are as follows.

(1) First, inspired by the work of Lin et al.[19] and Mahmood et al.[12], this study addresses the pain points of information expression in existing circular supplier selection research. We define a dedicated Euclidean distance measure specifically for PULTSFS. PULTSFS integrates the core advantages of probabilistic uncertain linguistic term sets (PULTS) and T-spherical fuzzy sets (TSFS). It combines three-dimensional expression of "membership-neutrality-non-membership" with probabilistic quantification. This approach accurately captures fuzziness, neutral cognition, and confidence differences in expert assessments. It prevents the loss of critical information and enhances the capability to characterize uncertain information.

(2) Second, we propose a FUCOM-ITARA combined weighting method. We draw upon the core logic of FUCOM and extend the ITARA method. We determine the indifference threshold through minimum Euclidean distances between pairwise alternatives. We then calculate deviation degrees

by combining these with distances to ideal solutions. This constructs a subjective-objective integrated weighting mechanism. It effectively enhances the scientific rigor of the weight determination process.

(3) Third, we propose a PULTSFS-CRADIS decision framework. We base this on the score function of PULTSFS and the proposed Euclidean distance. We improve the ideal solution approximation logic and ranking calculation process of CRADIS. We explicitly distinguish between cost-type and benefit-type criteria and complete normalization transformations. We calculate weighted distances between alternatives and positive/negative ideal solutions. We then combine utility values with ranking values to achieve rational alternative ranking. Finally, we construct a PULTSFS-based FUCOM-ITARA-CRADIS multi-criteria decision framework. This enables rational selection of circular suppliers under uncertain environments.

Section 2 defines the concept of PULTSFS and its basic operational rules. Section 3 elaborates the theoretical foundation and computational steps of the PULTSFS-based FUCOM-ITARA-CRADIS integrated decision-making model. Section 4 demonstrates the model's application through a circular supplier selection case. It conducts sensitivity analysis and comparative studies to validate the model's robustness and superiority. Section 5 summarizes research conclusions. It discusses research contributions, limitations, and future research directions.

2. preliminaries

2.1 PULTS and TSFS

Building upon uncertain linguistic variables (ULVs)[46]and probabilistic linguistic term sets (PLTS) [18], Lin et al.[19] proposed the concept of probabilistic uncertain linguistic term sets (PULTS). PULTS combines hesitant fuzzy linguistic term sets with probabilistic information. This combination maximally preserves the provided linguistic information.

Definition 1[18]. Let $S_{[0,k-1]} = \{s_0, s_1, \cdots s_{k-1}\}$be an LTS. A PLTS is defined as follows:

$$L(p) = \left\{s^{(t)}\left(p^{(t)}\right) \middle| s^{(t)} \in S_{[0,k-1]}, p^{(t)} \geq 0, t = 1,2,\ldots,\#T, \sum_{t=1}^{\#T} p^{(t)} \leq 1\right\} \tag{1}$$

Here, $s^{(t)}\left(p^{(t)}\right)$denotes the linguistic term s^{(t)} associated with probability $p^{(t)}$. #T represents the total number of distinct linguistic terms in L(p).

Definition 2[47]. A PULTS is defined as follows:

$$UL(p) = \left\{\langle [s_\alpha^{(t)}, s_\beta^{(t)}], p^{(t)}\rangle \middle| p^{(t)} \geq 0, t = 1,2,\ldots,\#T, \sum_{t=1}^{\#T} p^{(t)} \leq 1\right\} \tag{2}$$

Here, $\langle \left[s_\alpha^{(t)}, s_\beta^{(t)}\right], p^{(t)}\rangle$ constitute the t-th PUL element in UL(p). $s_\alpha^{(t)}, s_\beta^{(t)}$represent the lower and upper bound linguistic terms, respectively. $p^{(t)}$ denotes the corresponding probability.

Compared with traditional fuzzy sets and their variants, TSFS offers experts a broader expression space and greater flexibility in three dimensions. Mahmood et al. [12] provide the following definition:

Definition 3[12] . Let $X=\{x_1,x_2,\ldots,x_n\}$ be a universe of discourse. A TSFSA on X is then an object of the form:

$$A = \left\{\langle x_j, \left(\mu_A(x_j), \eta_A(x_j), \nu_A(x_j)\right)\rangle \,|x_j \in X\right\} \tag{3}$$

Here, $\mu_A(x_j), \eta_A(x_j), \nu_A(x_j): X \to [0,1]$denote the membership degree, neutrality degree, and non-membership degree of $x_j$to A, respectively. They satisfy the condition $0 \leq \left(\mu_A(x_j)\right)^q + \left(\eta_A(x_j)\right)^q + \left(\nu_A(x_j)\right)^q \leq 1 (q \geq 1)$.For computational convenience, a triplet $a_j = (\mu_j, \eta_j, \nu_j)$is called a T-spherical fuzzy number (TSFN). The parameter q can be set appropriately. A larger q value indicates weaker commitment strength. Conversely, a smaller q value indicates less hesitation and reduced uncertainty.

2.2 PULTSFS

Building upon the strengths of PULTS and TSFS, this study defines a novel probabilistic fuzzy set, namely PULTSFS. It allows experts to express evaluation information using multiple linguistic terms across three dimensions. It also incorporates the possibility of each linguistic term. The definition of PULTSFS is as follows:

Definition 4. Let$X$ be a universe of discourse, and let $S_{[0,k-1]}$ be an LTS. Then a PULTSFS $\tilde{A}(p)$is defined as:

$$\tilde{A}(p) = \{\langle x_\varsigma, \varphi_\varsigma(\hat{p})(x_\varsigma), \phi_\varsigma(\tilde{p})(x_\varsigma), \psi_\varsigma(\bar{p})(x_\varsigma)\rangle | x_\varsigma \in X\} \quad (4)$$

Where $\varphi_\varsigma(\hat{p})(x_\varsigma) = s_{\mu^L_{\varsigma(t)},\mu^U_{\varsigma(t)}}(p(t))$ with $s_{\mu^L_{\varsigma(t)},\mu^U_{\varsigma(t)}} \in S_{[0,k-1]}, p(t) \geq 0, \sum_{t=1}^{\#T} p(t) \leq 1$ denotes the membership degree of $x_\varsigma \in X$; $\phi_\varsigma(\tilde{p})(x_\varsigma) = s_{\eta^L_{\varsigma(r)},\eta^U_{\varsigma(r)}}(p(r))$ with $s_{\eta^L_{\varsigma(r)},\eta^U_{\varsigma(r)}} \in S_{[0,k-1]}, p(r) \geq 0, \sum_{r=1}^{\#R} p(r) \leq 1$ denotes the neutral degree of $x_\varsigma \in X$; $\psi_\varsigma(\bar{p})(x_\varsigma) = s_{\nu^L_{\varsigma(w)},\nu^U_{\varsigma(w)}}(p(w))$ with $s_{\nu^L_{\varsigma(w)},\nu^U_{\varsigma(w)}} \in S_{[0,k-1]}, p(w) \geq 0, \sum_{w=1}^{\#W} p(w) \leq 1$ denotes the non-membership degree of $x_\varsigma \in X$, and the associated probabilities are $\hat{p}_{(t)}, \tilde{p}_{(r)}$ and $\bar{p}_{(w)}$ respectively; for $x_\varsigma \in X$, they satisfy the condition $0 \leq \left(max_{t=1}^{\#T}\mu^U_{\varsigma(t)}\right)^q + \left(max_{r=1}^{\#R}\eta^U_{\varsigma(t)}\right)^q + \left(max_{w=1}^{\#W}\nu^U_{\varsigma(w)}\right)^q \leq k^q$(q⩾1).

If (X = {x}), then the PULTSFS ($\tilde{A}(p)$) degenerates into a PULTSFN ($\tilde{a}(p)$), i.e.,

$$\tilde{a}(p) = \langle\left\{\left[s_{\mu^L_{(t)}}, s_{\mu^U_{(t)}}\right] |\hat{p}_{(t)}\right\}, \left\{\left[s_{\eta^L_{(r)}}, s_{\eta^U_{(r)}}\right] |\tilde{p}_{(r)}\right\}, \left\{\left[s_{\nu^L_{(w)}}, s_{\nu^U_{(w)}}\right] |\bar{p}_{(w)}\right\}\rangle \quad (5)$$

where $s_{\mu^L_{(t)}}, s_{\mu^U_{(t)}}, s_{\eta^L_{(r)}}, s_{\eta^U_{(r)}}, s_{\nu^L_{(w)}}, s_{\nu^U_{(w)}} \in S_{[0,k-1]}$,$\sum_{t=1}^{\#T} \hat{p}_{(t)} \leq 1$,$\sum_{r=1}^{\#R} \tilde{p}_{(t)} \leq 1$,$\sum_{w=1}^{\#W} \bar{p}_{(w)} \leq 1$.

Note: For different values of the parameter q, the PULTSFS$\tilde{A}(p)$ can degenerate into different specific forms, as shown below:

When $(\phi_\varsigma(\tilde{p})(x_\varsigma) = 0)$, $(\tilde{A}(p))$degenerates into a probabilistic uncertain q-rung orthopair fuzzy set (PULqROFS)[21];

When $(q = 1)$and$(\phi_\varsigma(\tilde{p})(x_\varsigma) = 0)$, $\tilde{A}(p)$ degenerates into a probabilistic uncertain intuitionistic fuzzy set (PULIFS)[48];

When $(s_{\mu^L_{(t)}} = s_{\mu^U_{(t)}})$, $(s_{\eta^L_{(r)}} = s_{\eta^U_{(r)}})$and $(s_{\nu^L_{(w)}} = s_{\nu^U_{(w)}})$, $\tilde{A}(p)$degenerates into a probabilistic linguistic T-spherical fuzzy set (PLTSFS);

When $(\phi_\varsigma(\tilde{p})(x_\varsigma) = 0)$, $(s_{\mu^L_{(t)}} = s_{\mu^U_{(t)}})$ and $(s_{\nu^L_{(w)}} = s_{\nu^U_{(w)}})$, $\tilde{A}(p)$ degenerates into a probabilistic linguistic q-rung orthopair fuzzy set (PLq-ROFS)[49];

Obviously, the PULTSFS has strong generality, and under certain special circumstances, the PULTSFS can degenerate into some special forms.

Definition 5 [23]. Let any two PULTSFNs be $(\tilde{a}_1(p) = \left\{s_{\mu^L_{1(t)},\mu^U_{1(t)}}(p_{1(t)}), s_{\eta^L_{1(r)},\eta^U_{1(r)}}(p_{1(r)}), s_{\nu^L_{1(w)},\nu^U_{1(w)}}(p_{1(w)})\right\})$, $(\tilde{a}_2(p) = \left\{s_{\mu^L_{2(t)},\mu^U_{2(t)}}(p_{2(t)}), s_{\eta^L_{2(r)},\eta^U_{2(r)}}(p_{2(r)}), s_{\nu^L_{2(w)},\nu^U_{2(w)}}(p_{2(w)})\right\})$. For the convenience of calculation, they need to be normalized as follows:

Probability normalization: If $0 < \sum_{t=1}^{\#T} \hat{p}_{j(t)} < 1$ (taking the membership probability as an example, (j = 1,2)) , then $(\tilde{a}_j(p))$ is normalized to $(\tilde{a}_j(p^n))$, and the probability $\hat{p}^n_{j(t)} = \hat{p}_{j(t)}/\sum_{t=1}^{\#T} \hat{p}_{j(t)}$. The corresponding normalized PULTSFN can be described as

$$\tilde{a}_j(p^n) = \langle\left\{\left[s_{\mu^L_{j(t)}}, s_{\mu^U_{j(t)}}\right] |\hat{p}^n_{j(t)}\right\}, \left\{\left[s_{\eta^L_{j(r)}}, s_{\eta^U_{j(r)}}\right] |\tilde{p}^n_{j(r)}\right\}, \left\{\left[s_{\nu^L_{j(w)}}, s_{\nu^U_{j(w)}}\right] |\bar{p}^n_{j(w)}\right\}\rangle \quad (6)$$

Structure normalization: If $\#T_1 \neq \#T_2$ (taking the membership degree as an example), then it is necessary to add some linguistic terms to the one with fewer elements, with their probabilities set to 0, thereby obtaining the normalized PULTSFNs$(\tilde{a}_j(p^N))((j = 1,2))$.

Definition 6 [23]. Suppose $(S_{[0,k-1]})$ is an LTS, and $(\tilde{a}(p) = \left\{s_{\mu^L_{(t)},\mu^U_{(t)}}(p_{(t)}), s_{\eta^L_{(r)},\eta^U_{(r)}}(p_{(r)}), s_{\nu^L_{(w)},\nu^U_{(w)}}(p_{(w)})\right\})$ is a PULTSFN, where $(s_{\mu^L_{(t)}}, s_{\mu^U_{(t)}}, s_{\eta^L_{(r)}}, s_{\eta^U_{(r)}}, s_{\nu^L_{(w)}}, s_{\nu^U_{(w)}} \in S_{[0,k-1]}), (t = 1,2,\ldots,\#T; r = 1,2,\ldots,\#R; w = 1,2,\ldots,\#W)$ The score function of $\tilde{a}(p)$ is defined as

$$Sc(\tilde{a}(p)) = s_{\frac{1}{2}\left(1+\left(\frac{\sum_{t=1}^{\#T}\left(\frac{1}{2}\left(\mu^L_{(t)}+\mu^U_{(t)}\right)\hat{p}_{(t)}\right)}{k\sum_{t=1}^{\#T}\hat{p}_{(t)}}\right)^q-\left(\frac{\sum_{r=1}^{\#R}\left(\frac{1}{2}\left(\eta^L_{(t)}+\eta^U_{(t)}\right)\tilde{p}_{(r)}\right)}{k\sum_{r=1}^{\#R}\tilde{p}_{(r)}}\right)^q-\left(\frac{\sum_{w=1}^{\#W}\left(\frac{1}{2}\left(\nu^L_{(t)}+\nu^U_{(t)}\right)\bar{p}_{(w)}\right)}{k\sum_{w=1}^{\#W}\bar{p}_{(w)}}\right)^q\right)} \tag{7}$$

The accuracy function of \(\widetilde{a}(p)\) is defined as

$$Ac(\tilde{a}(p)) = s_{\left(\frac{\sum_{t=1}^{\#T}\left(\frac{1}{2}\left(\mu^L_{(t)}+\mu^U_{(t)}\right)\hat{p}_{(t)}\right)}{k\sum_{t=1}^{\#T}\hat{p}_{(t)}}\right)^q+\left(\frac{\sum_{r=1}^{\#R}\left(\frac{1}{2}\left(\eta^L_{(t)}+\eta^U_{(t)}\right)\tilde{p}_{(r)}\right)}{k\sum_{r=1}^{\#R}\tilde{p}_{(r)}}\right)^q+\left(\frac{\sum_{w=1}^{\#W}\left(\frac{1}{2}\left(\nu^L_{(t)}+\nu^U_{(t)}\right)\bar{p}_{(w)}\right)}{k\sum_{w=1}^{\#W}\bar{p}_{(w)}}\right)^q} \tag{8}$$

Definition 7. Suppose there are two arbitrary PULTSFNs$\tilde{a}_1(p)$ and$\tilde{a}_2(p)$. Then the comparison rules are as follows::

（1） if $Sc(\tilde{a}_1(p)) \succ Sc(\tilde{a}_2(p))$,then $\tilde{a}_1(p) \succ \tilde{a}_2(p)$;

（2） if $Sc(\tilde{a}_1(p)) \prec Sc(\tilde{a}_2(p))$,then $\tilde{a}_1(p) \prec \tilde{a}_2(p)$;

（3） if $Sc(\tilde{a}_1(p)) = Sc(\tilde{a}_2(p))$,then: i) if $Ac(\tilde{a}_1(p)) \succ Ac(\tilde{a}_2(p))$,then $\tilde{a}_1(p) \succ \tilde{a}_2(p)$; ii) if $Ac(\tilde{a}_1(p)) \prec Ac(\tilde{a}_2(p))$,then $\tilde{a}_1(p) \prec \tilde{a}_2(p)$; iii) if $Ac(\tilde{a}_1(p)) = Ac(\tilde{a}_2(p))$,then $\tilde{a}_1(p) \approx \tilde{a}_2(p)$;

Definition 8. Suppose $(S_{[0,k-1]})$ is an LTS. Let there be two PULTSFSs on (X)$x_{i\varsigma} \in X; \forall i = 1,2,\ldots,n$, $\tilde{A}(p) = \{\langle x_{i\varsigma}, \varphi_{A\varsigma}(\hat{p})(x_{i\varsigma}), \phi_{A\varsigma}(\tilde{p})(x_{i\varsigma}), \psi_{A\varsigma}(\bar{p})(x_{i\varsigma})\rangle | x_{i\varsigma} \in X\}$, $\tilde{B}(p) = \{\langle x_{i\varsigma}, \varphi_{B\varsigma}(\hat{p})(x_{i\varsigma}), \phi_{B\varsigma}(\tilde{p})(x_{i\varsigma}), \psi_{B\varsigma}(\bar{p})(x_{i\varsigma})\rangle | x_{i\varsigma} \in X\}$ Where $\varphi_{j\varsigma}(\hat{p})(x_{i\varsigma}) = \left\{\left[s_{\mu^L_{j\varsigma(t)}}, s_{\mu^U_{j\varsigma(t)}}\right](\hat{p}_{(t)}) \middle| s_{\mu^L_{j\varsigma(t)}}, s_{\mu^U_{j\varsigma(t)}} \in S_{[0,k-1]}, \hat{p}_{(t)} \geq 0, \sum_{t=1}^{\#T_j}\hat{p}_{(t)} \leq 1\right\}$denotes the membership degree of $x_{i\varsigma} \in X$; $\phi_{j\varsigma}(\tilde{p})(x_{i\varsigma}) = \left\{\left[s_{\eta^L_{j\varsigma(r)}}, s_{\eta^U_{j\varsigma(r)}}\right](\tilde{p}_{(r)}) \middle| s_{\eta^L_{j\varsigma(r)}}, s_{\eta^U_{j\varsigma(r)}} \in S_{[0,k-1]}, \tilde{p}_{(r)} \geq 0, \sum_{r=1}^{\#R_j}\tilde{p}_{(t)} \leq 1\right\}$ denotes the neutral degree of $x_{i\varsigma} \in X$; $\psi_{j\varsigma}(\bar{p})(x_{i\varsigma}) = \left\{\left[s_{\nu^L_{j\varsigma(w)}}, s_{\nu^U_{j\varsigma(w)}}\right](\bar{p}_{(w)}) \middle| s_{\nu^L_{j\varsigma(w)}}, s_{\nu^U_{j\varsigma(w)}} \in S_{[0,k-1]}, \bar{p}_{(w)} \geq 0, \sum_{w=1}^{\#W_j}\bar{p}_{(w)} \leq 1\right\}$denotes the non-membership degree of $x_{i\varsigma} \in X$, (j=A,B), $\#T_A$=$\#T_B$=$\#T$,$\#R_A$=$\#R_B$=$\#R$, $\#W_A$=$\#W_B$=$\#W$, and for the uncertain linguistic variable part, the function $I(\cdot)$ satisfies $I(s_\Theta) = \Theta$. Their normalized Euclidean distance $D_E\left(\tilde{A}(p), \tilde{B}(p)\right)$(α≥1) is expressed as follows:

$$D_H\left(\tilde{A}(p),\tilde{B}(p)\right) = \left\{\frac{1}{6n}\sum_{i=1}^{n}\left(\begin{array}{c}\frac{\sum_{t=1}^{\#T}\left(\begin{array}{c}\left[\left(I\left(s_{\mu^L_{A(t)}}\right)\hat{p}_{A(t)}\right)^q-\left(I\left(s_{\mu^L_{B(t)}}\right)\hat{p}_{B(t)}\right)^q\right]^2\\+\left[\left(I\left(s_{\mu^U_{A(t)}}\right)\hat{p}_{A(t)}\right)^q-\left(I\left(s_{\mu^U_{B(t)}}\right)\hat{p}_{B(t)}\right)^q\right]^2\end{array}\right)}{\#T}+\\\frac{\sum_{r=1}^{\#R}\left(\begin{array}{c}\left[\left(I\left(s_{\eta^L_{A(r)}}\right)\tilde{p}_{A(r)}\right)^q-\left(I\left(s_{\eta^L_{B(r)}}\right)\tilde{p}_{B(r)}\right)^q\right]^2\\+\left[\left(I\left(s_{\eta^U_{A(r)}}\right)\tilde{p}_{A(r)}\right)^q-\left(I\left(s_{\eta^U_{B(r)}}\right)\tilde{p}_{B(r)}\right)^q\right]^2\end{array}\right)}{\#R}+\\\frac{\sum_{w=1}^{\#W}\left(\begin{array}{c}\left[\left(I\left(s_{\nu^L_{A(w)}}\right)\bar{p}_{A(w)}\right)^q-\left(I\left(s_{\nu^L_{B(w)}}\right)\bar{p}_{B(w)}\right)^q\right]^2\\+\left[\left(I\left(s_{\nu^U_{A(w)}}\right)\bar{p}_{A(w)}\right)^q-\left(I\left(s_{\nu^U_{B(w)}}\right)\bar{p}_{B(w)}\right)^q\right]^2\end{array}\right)}{\#W}\end{array}\right)\right\}^{1/2} \tag{9}$$

3. A multi-criteria decision-making model based on PULTSFS FUCOM-ITARA-CRADIS

In a general MADM problem, decision-makers evaluate a finite set of alternatives: $\mathcal{H} = \{\mathscr{k}_1, \mathscr{k}_2, \ldots, \mathscr{k}_m\}$,consisting of m alternatives, where$\mathscr{k}_i$is the i-th alternative. There is a corresponding set of attributes: $\mathcal{A} = \{a_1, a_2, \ldots, a_n\}$, consisting of n attributes, wherea_j is the j-th attribute. Each attribute is assigned a relative importance represented by a weight vector: $\boldsymbol{w} = (w_1, w_2, \ldots, w_n)^T$, where$w_j \geq 0$ and $\sum_{j=1}^{n} w_j = 1$. The evaluation information provided by experts for each alternative under each attribute is expressed using PULTSFNs. These evaluation results form a decision matrix $\widetilde{\mathcal{D}} = \left[\tilde{d}_{ij}(p)\right]_{m\times n}$with PULTSF information, where $\tilde{d}_{ij}(p) = \langle\left\{\left[s_{\mu^L_{ij(t)}}, s_{\mu^U_{ij(t)}}\right] \middle| \hat{p}_{ij(t)}\right\}, \left\{\left[s_{\eta^L_{ij(r)}}, s_{\eta^U_{ij(r)}}\right] \middle| \tilde{p}_{ij(r)}\right\}, \left\{\left[s_{\nu^L_{ij(w)}}, s_{\nu^U_{ij(w)}}\right] \middle| \bar{p}_{ij(w)}\right\}\rangle$(*i*=1, 2,…,*m*; *j*=1, 2,…,*n*; *t*=1, 2,…,#*T*; *r*=1, 2, …,#*R*; *w*=1, 2, …,#*W*).

3.1 Determination of combined criteria weights based on PULTSF FUCOM-ITARA

Step 1. Normalize and standardize the individual initial PULTSF decision matrices. According to Definition 5, each initial PULTSF decision matrix is normalized so that the PULTSFNs in the matrix are consistent in form. In addition, for the two types of attributes, namely cost-type and benefit-type, they need to be converted to the same type to avoid adverse consequences. Formula (17) is usually used to convert cost-type attributes into benefit-type ones. After conversion, the individual standardized PULTSF decision matrix is expressed as $\widetilde{\mathcal{D}} = \left[\tilde{d}_{ij}(p)\right]_{m\times n}$

$$\tilde{d}_{ij}(p) = \begin{cases} \tilde{d}^{(\varepsilon)}_{ij}(p) = \left(\left\{\begin{bmatrix} s_{\mu^{L(\varepsilon)}_{ij(t)}}, \\ s_{\mu^{U(\varepsilon)}_{ij(t)}} \end{bmatrix} \middle| \hat{p}^{(\varepsilon)}_{ij(t)}\right\}, \left\{\begin{bmatrix} s_{\eta^{L(\varepsilon)}_{ij(t)}}, \\ s_{\eta^{U(\varepsilon)}_{ij(t)}} \end{bmatrix} \middle| \tilde{p}^{(\varepsilon)}_{ij(r)}\right\}, \left\{\begin{bmatrix} s_{\nu^{L(\varepsilon)}_{ij(t)}}, \\ s_{\nu^{U(\varepsilon)}_{ij(t)}} \end{bmatrix} \middle| \bar{p}^{(\varepsilon)}_{ij(w)}\right\}\right), j \in J_1 \\ \left(\tilde{d}^{(\varepsilon)}_{ij}(p)\right)^c = \left(\left\{\begin{bmatrix} s_{\nu^{L(\varepsilon)}_{ij(t)}}, \\ s_{\nu^{U(\varepsilon)}_{ij(t)}} \end{bmatrix} \middle| \bar{p}^{(\varepsilon)}_{ij(w)}\right\}, \left\{\begin{bmatrix} s_{\eta^{L(\varepsilon)}_{ij(t)}}, \\ s_{\eta^{U(\varepsilon)}_{ij(t)}} \end{bmatrix} \middle| \tilde{p}^{(\varepsilon)}_{ij(r)}\right\}, \left\{\begin{bmatrix} s_{\mu^{L(\varepsilon)}_{ij(t)}}, \\ s_{\mu^{U(\varepsilon)}_{ij(t)}} \end{bmatrix} \middle| \hat{p}^{(\varepsilon)}_{ij(t)}\right\}\right), j \in J_2 \end{cases} \quad (10)$$

where$(J_1)$and(J_2) denote the benefit-type and cost-type attributes, respectively.

Step 2. Determination method of subjective weights for attributes based on FUCOM.

In the framework of multi-criteria decision making, the determination of relative weights of criteria is considered a subjective issue. Since weight coefficients have a critical impact on the solutions of certain methods, this process plays a significant and crucial role in the final outcomes of the multi-criteria decision making environment. This paper adopts the Full Consistency Method (FUCOM) to calculate the objective weights of criteria. Under the premise of determining the hierarchical structure and jointly satisfying the comparative consistency conditions, this method can accurately measure the ratings of the weight coefficients of criteria. The steps for determining the criteria weights using the Full Consistency Method (FUCOM) are as follows:

Step 2.1. First, experts evaluate the importance of each criterion, and the preference order is obtained based on the importance of the criteria from high to low. Therefore, the ranking of criteria derived from the expected values of the weight coefficients can be expressed as:

$$\mathrm{a}_{j(1)} > \mathrm{a}_{j(2)} > \mathrm{a}_{j(3)} > \ldots > \mathrm{a}_{j(\sigma)} \quad (11)$$

where (σ) denotes the rank of the observed criterion.

Step 2.2. Determine the comparative priority of the evaluation criteria $((\theta\_\{\sigma/(\sigma+1)\}; \sigma = 1,2,3 \ldots n))$, where the comparative priority represents the degree of advantage of the criterion ranked (σ) over the criterion ranked $(\sigma + 1)$. Thus, the comparative priority vector of the evaluation criteria is obtained as:

$$\Phi = \left(\theta_{1/2}, \theta_{2/3}, \ldots, \theta_{\sigma/(\sigma+1)}\right) \quad (12)$$

where$(\theta_{\sigma/(\sigma+1)})$denotes the importance (priority) of the criterion ranked (σ) relative to the criterion ranked $(\sigma + 1)$. This paper adopts a predetermined scale for comparing criteria: based on the ranking results from Step 2.1, the decision-maker uses the criterion ranked first as the benchmark (importance = 1) and applies a predetermined scale (e.g., 1–9 scale) to determine the priority (w_σ) of all criteria, requiring a total of $(\mathrm{n} - 1)$ comparisons.

Step 2.3. Calculate the weight coefficient results of the evaluation criteria $w = \left(\mathrm{w}_1, \mathrm{w}_2, \ldots, \mathrm{w}_\mathrm{j}\right)^\mathrm{T}$.following two constraints need to be satisfied for the final weight coefficient results:

（1）The comparative preferences among the considered criteria are consistent with the ratios of the weight coefficients, i.e., the following condition must be satisfied:

$$\frac{w_\sigma}{w_{\sigma+1}} = \theta_{\sigma/(\sigma+1)} \tag{13}$$

（2）In addition to formula (20), the overall degree of the weight coefficients must satisfy the mathematical transitivity condition, i.e., $\theta_{\sigma/(\sigma+1)} \times \theta_{(\sigma+1)/(\sigma+2)} = \theta_{\sigma/(\sigma+2)}$.Since $\theta_{\sigma/(\sigma+1)} = \frac{w_\sigma}{w_{\sigma+1}}$ and $\theta_{(\sigma+1)/(\sigma+2)} = \frac{w_{\sigma+1}}{w_{\sigma+2}}$,it follows that $\frac{w_\sigma}{w_{\sigma+2}} = \frac{w_\sigma}{w_{\sigma+1}} \times \frac{w_{\sigma+1}}{w_{\sigma+2}}$; thus, the final degree of the weight coefficients of the evaluation criteria must satisfy another constraint condition, namely:

$$\frac{w_\sigma}{w_{\sigma+2}} = \theta_{\sigma/(\sigma+1)} \times \theta_{(\sigma+1)/(\sigma+2)} \tag{14}$$

It should be noted that both of the above conditions need to be satisfied simultaneously in order to achieve the minimum "deviation from full consistency (DFC)" ((X)). To achieve this goal, the values of the weight coefficients $w = \left(w_1, w_2, \ldots, w_j\right)^T$ must satisfy the conditions $|\theta_{\sigma/(\sigma+1)} - \frac{w_\sigma}{w_{\sigma+1}}| \leq X$ and $\left|\frac{w_\sigma}{w_{\sigma+2}} - \theta_{\sigma/(\sigma+1)} \times \theta_{(\sigma+1)/(\sigma+2)}\right| \leq X$, while minimizing X (i.e., maximizing consistency).

Based on the above constraints and settings, the following objective model is proposed for calculating the final degree of the weight coefficients of the evaluation criteria:

$$\begin{aligned} & Min\, X \\ & s.t. \\ & \quad |\theta_{\sigma/(\sigma+1)} - \frac{w_\sigma}{w_{\sigma+1}}| \leq X, \ \ \forall\sigma \\ & \quad \left|\frac{w_\sigma}{w_{\sigma+2}} - \theta_{\sigma/(\sigma+1)} \times \theta_{(\sigma+1)/(\sigma+2)}\right| \leq X, \ \forall\sigma \\ & w_j \geq 0, \qquad \forall\sigma, \\ & \sum_{j=1}^{n} \mathrm{w_j} = 1. \end{aligned} \tag{15}$$

By solving Equation (15), the final weights of the evaluation criteria $w_f = \left(w_{f1}, w_{f2}, \ldots, w_{fj}\right)^T$ and the DFC (X) of the results can be obtained.

Step 3. Determination method of objective weights for attributes based on ITARA.

In 2019, some scholars proposed the ITARA method based on the concepts of indifference threshold and dispersion logic[32]. It is used to determine the objective weights of attributes in MCDM problems, and can obtain attribute weights from the data in the decision matrix rather than requiring experts to provide information about the attributes. Although the ITARA method has this advantage, it also has some drawbacks. For example, the indifference threshold in the ITARA method is not quantified but is subjectively determined by decision-makers; the process of precise quantization leads to partial loss of fuzzy and uncertain information, etc. Therefore, the ITARA method is extended and improved in the probabilistic uncertain linguistic T-spherical fuzzy environment to determine the objective weights of attributes. The specific procedure is as follows.

Step 3.1. Determine the PULTSF positive ideal solution in the PULTSF matrix $\widetilde{\mathcal{D}} = \left[\tilde{d}_{ij}(p)\right]_{m\times n}$, denoted as $d_{ij}{}^{+} = \left\{\tilde{r}_1^{+}(p), \tilde{r}_2^{+}(p), \cdots, \tilde{r}_j^{+}(p), \cdots, \tilde{r}_n^{+}(p)\right\}$, where $\tilde{Q}^{+}(p) = \left(\tilde{r}_{ij}(p) \left| \max_i \left\{Sc\left(\tilde{r}_{ij}(p)\right)\right\}\right.\right)$.

Step 3.2. Calculate the indifference threshold for each attribute. Using Formula (16), the minimum Euclidean distance between pairs of alternatives is adopted as the indifference threshold, denoted as ε_j.

$$\varepsilon_j = \min D_H\left(d_{ij}, d_{kj}\right) | i, k = 1,2, \ldots m; i \neq k \tag{16}$$

Step 3.3. Calculate the effective deviation. For the evaluation value of each alternative h_m under attribute a_n, the deviation relative to the positive ideal solution is calculated by Formula (17) as the basic deviation g_{ij}. Then, Formula (18) is used to filter out the effective deviation r_{ij}.

$$g_{ij} = D_H\left(d_{ij}, {d_{ij}}^{+}\right) \tag{17}$$

$$r_{ij} = \begin{cases} g_{ij} - \varepsilon_j & g_{ij} > \varepsilon_j \\ 0 & g_{ij} \leq \varepsilon_j \end{cases} \tag{18}$$

Step 3.4. Calculate the attribute weights. The effective deviations under each alternative are summed, and finally normalized by Formula (19) to obtain the objective weights $w_i = \left(w_{i1}, w_{i2}, \ldots, w_{ij}\right)^T$

$$w_i = \frac{\Sigma_{i=1}^{n} r_{ij}}{\Sigma_{i=1}^{n} \Sigma_{j=1}^{n} r_{ij}} \tag{19}$$

Step 4. Calculate the combined weight of attributes w_j according to Formula (20).

$$w_j = \frac{w_f \times w_i}{\sum_{f=1}^{n} w_f \times \Sigma_{i=1}^{n} w_i} \tag{20}$$

3.2 PULTSF-CRADIS method

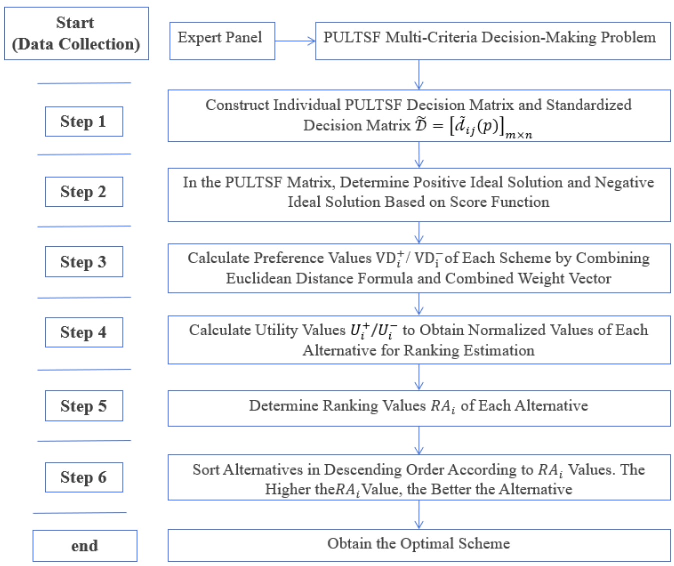

Fig 1. Decision-making flowchart of the PULTSF-CRADIS method

In the framework of multi-criteria decision making, the ranking of alternatives is a core link that determines the quality of the final decision, and the suitability of the ranking method has a critical impact on the scientific validity of the decision results. The CRADIS method proposed by Puška et al.[30] in 2021 can efficiently rank alternatives in MCDM problems, integrating the core advantages of ARAS[37], MARCOS[38] and TOPSIS[39] methods. Addressing the issue of information representation in existing studies on circular supplier selection, this method cannot handle probabilistic uncertain linguistic T-spherical fuzzy information. Therefore, this paper extends the CRADIS method to this fuzzy environment for improvement and optimization, and uses it for the ranking and selection of circular supplier alternatives. The specific procedure is as follows. The decision-making flowchart is shown in Figure 1.

Step 1. Normalize and standardize the individual initial PULTSF decision matrices. According to Definition 5, each initial PULTSF decision matrix is normalized so that the PULTSFNs in the matrix are consistent in form. In addition, for the two types of attributes, namely cost-type and benefit-type, they need to be converted to the same type to avoid adverse consequences. Formula (12) is usually used to convert cost-type attributes into benefit-type ones. After conversion, the individual standardized PULTSF decision matrix is expressed as $\widetilde{\mathcal{D}} = \left[\tilde{d}_{ij}(p)\right]_{m\times n}$.

where J_1 and J_2 denote the benefit-type and cost-type attributes, respectively.

Step 2. Determine the PULTSF positive ideal solution in the PULTSF matrix $\widetilde{\mathcal{D}} = \left[\tilde{d}_{ij}(p)\right]_{m\times n}$, denoted as $\tilde{Q}^+(p) = \{\tilde{r}_1^+(p), \tilde{r}_2^+(p), \cdots, \tilde{r}_j^+(p), \cdots, \tilde{r}_n^+(p)\}$,where $\tilde{Q}^+(p) = \left(\tilde{r}_{ij}(p) \left| \max_i \left\{Sc\left(\tilde{r}_{ij}(p)\right)\right\}\right.\right)$. Similarly, the PULTSF negative ideal solution is denoted as $\tilde{Q}^-(p) = \{\tilde{r}_1^-(p), \tilde{r}_2^-(p), \cdots, \tilde{r}_j^-(p), \cdots, \tilde{r}_n^-(p)\}$,where $\tilde{Q}^-(p) = \left(\tilde{r}_{ij}(p) \left| \min_i \left\{Sc\left(\tilde{r}_{ij}(p)\right)\right\}\right.\right)$.

Step 3. Combining the Euclidean distance formula (Formula (9)) with the combined weight vector $w = (w_1, w_2, \ldots, w_n)^T$ from Section 3.1, the preference values of each alternative are calculated using Formulas (21)-(22).

$$VD_i^+ = \sum_{j=1}^{n} W_j D_H\left(\tilde{r}_{ij}(p), \tilde{Q}^+(p)\right) \quad (21)$$

$$VD_i^- = \sum_{j=1}^{n} W_j D_H\left(\tilde{r}_{ij}(p), \tilde{Q}^-(p)\right) \quad (22)$$

Step 4. The utility values are calculated using Formulas (23)-(24), and the normalized value for each alternative is obtained for ranking evaluation.

$$U_i^+ = \frac{min\ (VD_i^+)}{VD_i^+} \quad (23)$$

$$U_i^- = \frac{VD_i^-}{max\ (VD_i^-)} \quad (24)$$

where $max\ (VD_i^-)$ is the maximum value among the different VD_i^- values, and $min\ (VD_i^+)$ is the minimum value among the different VD_i^+ values.

Step 5. The ranking value of each alternative is determined using Formula (25).

$$RA_i = \frac{U_i^+ + U_i^-}{2} \quad (25)$$

Step 6. The alternatives are sorted in descending order according to their RA_i values, i.e., the alternative with a higher RA_i value is better, and the most ideal alternative is determined.

4 Case analysis of circular supplier selection

This section discusses a case study of the leading new energy battery manufacturing enterprise in China, "Huayu New Energy Group (HYNE)", to verify the practicality of the proposed PULTSF-FUCOM-CRADIS tool. Huayu New Energy Group is a holding group focusing on the entire industrial chain of new energy storage and power batteries. It currently holds controlling stakes in 12 core subsidiaries, covering key areas such as battery research and development, material production, battery assembly, and recycling. With the advancement of global carbon peaking and carbon neutrality goals and the rapid development of the new energy industry, the recycling of waste power batteries has become a critical issue for the sustainable development of the industry. Its subsidiary, "Green Cycle Technology", specializes in the recycling of battery materials. In this case study, Green Cycle Technology needs to find sustainable circular suppliers for the precursor of the core NCM811 cathode material for ternary lithium batteries, ensuring the green sustainability of the supply chain.

To evaluate the indicators and alternatives, a panel consisting of four decision-making experts (E_1, E_2, E_3, E_4) was formed to carry out the sustainable circular supplier selection. These decision-making experts come from different fields, have over 20 years of experience, are proficient in decision-making practices, and possess strong professional competence in various circular supply chain activities. The decision-making expert panel reviewed the information on indicators for sustainable circular supplier evaluation, combined literature review and expert opinions, and collected indicators that may affect the evaluation of sustainable circular suppliers. Finally, four criteria—circular（a_1）, environmental (a_2), economic (a_3), and social (a_4)—were all defined as benefit-type indicators for evaluation. This case study considers four suppliers, namely Sustainable Circular Supplier 1 （h_1）, Sustainable Circular Supplier 2 （h_2）, Sustainable Circular Supplier 3 （h_3 ）, and Sustainable Circular Supplier 4（h_4）. To accurately describe the information in the evaluation of the four criteria, the linguistic term set $S_{[0,6]} = \{s_0, s_1, \cdots, s_6\} = \{\text{very low, low, moderately low, medium, moderately high, high, very high}\}$ was used. The expert group was authorized to use PULTSFNs to describe the evaluation information. The evaluation data provided by the expert group are shown in Table 1.

Table 1. Evaluation information provided by experts

	a_1	a_2	a_3	a_4
h_1	$<\{[s_2,s_3]\Vert 0.5,[s_3,s_4]\Vert 0.5\},$ $\{[s_1,s_2]\Vert 0.3,[s_2,s_3]\Vert 0.7\},$ $\{[s_2,s_3]\Vert 0.5,[s_3,s_4]\Vert 0.5\}>$	$<\{s_3\vert 0.3,[s_3,s_4]\Vert 0.7\},$ $\{[s_1,s_2]\Vert 0.6,[s_2,s_3]\Vert 0.4\},$ $\{[s_1,s_2]\Vert 0.2,[s_2,s_3]\Vert 0.8\}>$	$<\{[s_4,s_5]\Vert 0.7,[s_5,s_6]\Vert 0.3\}$ $\{[s_2,s_3]\Vert 0.5,[s_3,s_4]\Vert 0.5\},$ $\{[s_1,s_2]\Vert 0.6,[s_2,s_3]\Vert 0.4\}>$	$<\{[s_3,s_4]\Vert 0.4,[s_4,s_5]\Vert 0.6\}$ $\{[s_0,s_1]\Vert 0.7,[s_1,s_2]\Vert 0.3\},$ $\{[s_2,s_3]\Vert 0.6,[s_3,s_4]\Vert 0.4\}>$
h_2	$<\{[s_3,s_4]\Vert 1\},$ $\{[s_0,s_1]\Vert 0.2,[s_1,s_2]\Vert 0.8\},$ $\{[s_3,s_4]\Vert 0.3,[s_4,s_5]\Vert 0.7\}>$	$<\{[s_3,s_4]\Vert 0.2,[s_4,s_5]\Vert 0.8\}$ $\{[s_2,s_3]\Vert 0.6,[s_3,s_4]\Vert 0.4\},$ $\{[s_2,s_3]\Vert 0.3,[s_3,s_4]\Vert 0.7\}>$	$<\{s_3\vert 0.7,[s_3,s_4]\Vert 0.3\},$ $\{[s_1,s_2]\Vert 0.6,[s_2,s_3]\Vert 0.4\},$ $\{[s_2,s_3]\Vert 0.5,[s_3,s_4]\Vert 0.5\}>$	$<\{[s_1,s_2]\Vert 0.3,[s_2,s_3]\Vert 0.7\}$ $\{[s_1,s_2]\Vert 0.5,[s_2,s_3]\Vert 0.5\},$ $\{[s_3,s_4]\Vert 0.8,[s_4,s_5]\Vert 0.2\}>$
h_3	$<\{[s_3,s_4]\Vert 0.4,[s_4,s_5]\Vert 0.6\},$ $\{[s_1,s_2]\Vert 0.5,[s_2,s_3]\Vert 0.5\},$ $\{[s_1,s_2]\Vert 0.7,[s_2,s_3]\Vert 0.3\}>$	$<\{[s_3,s_4]\Vert 0.7,[s_4,s_5]\Vert 0.3\}$ $\{[s_0,s_1]\Vert 0.8,[s_1,s_2]\Vert 0.2\},$ $\{[s_1,s_2]\Vert 0.6,[s_2,s_3]\Vert 0.4\}>$	$<\{[s_1,s_2]\Vert 0.3,[s_2,s_3]\Vert 0.7\},$ $\{[s_1,s_2]\Vert 0.8,[s_2,s_3]\Vert 0.2\},$ $\{[s_3,s_4]\Vert 0.4,[s_4,s_5]\Vert 0.6\}>$	$<\{[s_2,s_3]\Vert 0.6,[s_3,s_4]\Vert 0.4\},$ $\{[s_0,s_1]\Vert 0.7,[s_1,s_2]\Vert 0.3\},$ $\{[s_2,s_3]\Vert 0.4,[s_3,s_4]\Vert 0.6\}>$
h_4	$<\{[s_2,s_3]\Vert 0.6,[s_3,s_4]\Vert 0.4\},$ $\{[s_1,s_2]\Vert 0.6,[s_2,s_3]\Vert 0.4\},$ $\{[s_3,s_4]\Vert 0.5,s_4\vert 0.5\}>$	$<\{[s_0,s_1]\Vert 0.4,[s_1,s_2]\Vert 0.6\}$ $\{[s_1,s_2]\Vert 0.6,[s_2,s_3]\Vert 0.4\},$ $\{[s_3,s_4]\Vert 0.5,[s_4,s_5]\Vert 0.5\}>$	$<\{s_4\vert 0.4,[s_4,s_5]\Vert 0.6\},$ $\{[s_1,s_2]\Vert 0.7,[s_2,s_3]\Vert 0.3\},$ $\{[s_2,s_3]\Vert 0.5,[s_3,s_4]\Vert 0.5\}>$	$<\{[s_2,s_3]\Vert 0.3,[s_3,s_4]\Vert 0.7\}$ $\{[s_1,s_2]\Vert 0.4,[s_2,s_3]\Vert 0.6\},$ $\{[s_1,s_2]\Vert 0.8,[s_2,s_3]\Vert 0.2\}>$

4.1 Application process

Step 1. Using Definition 5 and Formula (12), the standardized PULTSF decision matrix $\widetilde{\mathcal{D}} = \left[\widetilde{\tilde{d}_{ij}(p)}\right]_{m\times n}$is obtained, as shown in Table 2.

Table 2. Standardized PULTSF decision matrix

	a_1	a_2	a_3	a_4
h_1	$<\{[s_2,s_3]\Vert 0.5,[s_3,s_4]\Vert 0.5\},$ $\{[s_1,s_2]\Vert 0.3,[s_2,s_3]\Vert 0.7\},$ $\{[s2,s3]\Vert 0.5,[s3,s_4]\Vert 0.5\}>$	$<\{[s_1,s_2]\Vert 0.2,[s_2,s_3]\Vert 0.8\},$ $\{[s_1,s_2]\Vert 0.6,[s_2,s_3]\Vert 0.4\},$ $\{[s_3,s_3]\Vert 0.3,[s_3,s_4]\Vert 0.7\}>$	$<\{[s_4,s_5]\Vert 0.7,[s_5,s_6]\Vert 0.3\},$ $\{[s_2,s_3]\Vert 0.5,[s_3,s_4]\Vert 0.5\},$ $\{[s_1,s_2]\Vert 0.6,[s_2,s_3]\Vert 0.4\}>$	$<\{[s_3,s_4]\Vert 0.4,[s_4,s_5]\Vert 0.6\},$ $\{[s_0,s_1]\Vert 0.7,[s_1,s_2]\Vert 0.3\},$ $\{[s_2,s_3]\Vert 0.6,[s_3,s_4]\Vert 0.4\}>$
h_2	$<\{[s_3,s_4]\Vert 0.5,[s_3,s_4]\Vert 0.5\},$ $\{[s_0,s_1]\Vert 0.2,[s_1,s_2]\Vert 0.8\},$ $\{[s_3,s_4]\Vert 0.3,[s_4,s_5]\Vert 0.7\}>$	$<\{[s_2,s_3]\Vert 0.3,[s_3,s_4]\Vert 0.7\},$ $\{[s_2,s_3]\Vert 0.6,[s_3,s_4]\Vert 0.4\},$ $\{[s_3,s_4]\Vert 0.2,[s_4,s_5]\Vert 0.8\}>$	$<\{[s_3,s_3]\Vert 0.7,[s_3,s_4]\Vert 0.3\},$ $\{[s_1,s_2]\Vert 0.6,[s_2,s_3]\Vert 0.4\},$ $\{[s_2,s_3]\Vert 0.5,[s_3,s_4]\Vert 0.5\}>$	$<\{[s_1,s_2]\Vert 0.3,[s_2,s_3]\Vert 0.7\},$ $\{[s_1,s_2]\Vert 0.5,[s_2,s_3]\Vert 0.5\},$ $\{[s_3,s_4]\Vert 0.8,[s_4,s_5]\Vert 0.2\}>$
h_3	$<\{[s_3,s_4]\Vert 0.4,[s_4,s_5]\Vert 0.6\},$ $\{[s_1,s_2]\Vert 0.5,[s_2,s_3]\Vert 0.5\},$ $\{[s_1,s_2]\Vert 0.7,[s_2,s_3]\Vert 0.3\}>$	$<\{[s_1,s_2]\Vert 0.6,[s_2,s_3]\Vert 0.4\},$ $\{[s_0,s_1]\Vert 0.8,[s_1,s_2]\Vert 0.2\},$ $\{[s_3,s_4]\Vert 0.7,[s_4,s_5]\Vert 0.3\}>$	$<\{[s_1,s_2]\Vert 0.3,[s_2,s_3]\Vert 0.7\},$ $\{[s_1,s_2]\Vert 0.8,[s_2,s_3]\Vert 0.2\},$ $\{[s_3,s_4]\Vert 0.4,[s_4,s_5]\Vert 0.6\}>$	$<\{[s_2,s_3]\Vert 0.6,[s_3,s_4]\Vert 0.4\},$ $\{[s_0,s_1]\Vert 0.7,[s_1,s_2]\Vert 0.3\},$ $\{[s_2,s_3]\Vert 0.4,[s_3,s_4]\Vert 0.6\}>$
h_4	$<\{[s_2,s_3]\Vert 0.6,[s_3,s_4]\Vert 0.4\},$ $\{[s_1,s_2]\Vert 0.6,[s_2,s_3]\Vert 0.4\},$ $\{[s_3,s_4]\Vert 0.5,[s_4,s_4]\Vert 0.5\}>$	$<\{[s_3,s_4]\Vert 0.5,[s_4,s_5]\Vert 0.5\},$ $\{[s_1,s_2]\Vert 0.6,[s_2,s_3]\Vert 0.4\},$ $\{[s_0,s_1]\Vert 0.4,[s_1,s_2]\Vert 0.6\}>$	$<\{[s_4,s_4]\Vert 0.4,[s_4,s_5]\Vert 0.6\},$ $\{[s_1,s_2]\Vert 0.7,[s_2,s_3]\Vert 0.3\},$ $\{[s_2,s_3]\Vert 0.5,[s_3,s_4]\Vert 0.5\}>$	$<\{[s_2,s_3]\Vert 0.3,[s_3,s_4]\Vert 0.7\},$ $\{[s_1,s_2]\Vert 0.4,[s_2,s_3]\Vert 0.6\},$ $\{[s_1,s_2]\Vert 0.8,[s_2,s_3]\Vert 0.2\}>$

Step 2. Determination of attribute weights based on PULTSF-FUCOM.

Step 2.1.Based on the experts' assessment of the importance of each criterion and reference[50], the ranking of the importance of the criteria is: Environment > Circular > Social > Economy, i.e., $\mathrm{a}_2 > \mathrm{a}_1 > \mathrm{a}_4 > \mathrm{a}_3$.

Step 2.2. Determine the comparative priority of the evaluation criteria. Using the criterion ranked first C_2 as the benchmark (importance = 1), the importance of the other criteria is quantified using a 1–9 scale, based on expert evaluation and reference[51], as shown in Table 3.

Table 3. Priority of criteria

criteria	a_2	a_1	a_4	a_3

w_{σ}	1	2.1	3	3

Based on the priority values, the comparative priority is calculated as: $\theta_{2/1}$ = 2.1/1 = 2.1, $\theta_{1/4}$= 3/2.1 = 1.43, $\theta_{4/3}$ = 3/3 = 1.

Step 2.3. Calculate the weight coefficient results of the evaluation criteria.

The final values of the weight coefficients should satisfy the following two conditions:

（1）The final conditions for the weight coefficients should satisfy$\frac{w_2}{w_1}=2.1$，$\frac{w_1}{w_4}=1.43$，$\frac{w_4}{w_3}=1$；

（2）Based on the transitivity condition, it should satisfy: $\frac{w_2}{w_4}=2.1\times1.43=3$，$\frac{w_1}{w_3}=1.43\times1=1.43$. By applying Formula (15), the final model for determining the weight coefficients can be defined as:

$$Min\ X$$
$$s.t.\begin{cases}\left|\frac{w_2}{w_1}-2.1\right|\le X,\left|\frac{w_1}{w_4}-1.43\right|\le X,\left|\frac{w_4}{w_3}-1\right|\le X,\\ \left|\frac{w_2}{w_4}-3\right|\le X,\left|\frac{w_1}{w_3}-1.43\right|\le X,\\ \sum_{j=1}^{n}\mathrm{w_j}=1,w_j\ge0,\forall\sigma\end{cases} \tag{26}$$

Solving this model yields DFC（X=0.00）, indicating that the weight results have no deviation and fully satisfy the consistency requirement. The final subjective weight coefficients are obtained as: $w_f=(0.222,0.466,0.156,0.156)^{\mathrm{T}}$.

Step 3. Determination of objective attribute weights based on PULTSF-ITARA.

Step 3.1. Obtain the positive ideal solution according to the score function, following the same procedure as in Step 5.

Step 3.2. Calculate the indifference threshold for each attribute according to Formula (16). The indifference thresholds for the four criteria are ε_j={2.389，4.935，5.429，2.205}.

Step 3.3. The basic deviation and effective deviation are obtained according to Formulas (17)-(18), and the results are shown in Table 4.

Table 4. Deviation degree of each alternative

Alternatives	g_{ij}				r_{ij}			
	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
h_1	4.887	4.935	0.000	0.000	2.498	0.000	0.000	0.000
h_2	10.685	14.532	7.683	7.499	8.295	9.597	2.254	5.294
h_3	0.000	6.031	11.563	5.787	0.000	1.095	6.134	3.582
h_4	5.873	0.000	10.195	2.205	3.483	0.000	4.766	0.000

Step 3.4. After normalization, t.he objective attribute weights are obtained. $w_f=(0.304\ ,0.227\ ,0.280\ ,0.189\ )^{\mathrm{T}}$.

Step 4. Combining the subjective and objective weights, the final criteria weights are obtained as $w_j=(0.273,0.430,0.177,0.120)^{\mathrm{T}}$.

Step 5. Determine the PULTSF positive and negative ideal solutions.

$$PIS=\begin{pmatrix}\beth_1: & \langle\{[s_3,s_4]|0.4,[s_4,s_5]|0.6\},\{[s_1,s_2]|0.5,[s_2,s_3]|0.5\},\{[s_1,s_2]|0.7,[s_2,s_3]|0.3\}\rangle\\ \beth_2: & \langle\{[s_3,s_4]|0.5,[s_4,s_5]|0.5\},\{[s_1,s_2]|0.6,[s_2,s_3]|0.4\},\{[s_0,s_1]|0.4,[s_1,s_2]|0.6\}\rangle\\ \beth_3: & \langle\{[s_4,s_5]|0.7,[s_5,s_6]|0.3\},\{[s_2,s_3]|0.5,[s_3,s_4]|0.5\},\{[s_1,s_2]|0.6,[s_2,s_3]|0.4\}\rangle\\ \beth_4: & \langle\{[s_3,s_4]|0.4,[s_4,s_5]|0.6\},\{[s_0,s_1]|0.7,[s_1,s_2]|0.3\},\{[s_2,s_3]|0.6,[s_3,s_4]|0.4\}\rangle\end{pmatrix}$$

$$NIS=\begin{pmatrix}\beth_1: & \langle\{[s_2,s_3]|0.6,[s_3,s_4]|0.4\},\{[s_1,s_2]|0.6,[s_2,s_3]|0.4\},\{[s_3,s_4]|0.5,[s_4,s_4]|0.5\}\rangle\\ \beth_2: & \langle\{[s_2,s_3]|0.3,[s_3,s_4]|0.7\},\{[s_2,s_3]|0.6,[s_3,s_4]|0.4\},\{[s_3,s_4]|0.2,[s_4,s_5]|0.8\}\rangle\\ \beth_3: & \langle\{[s_1,s_2]|0.3,[s_2,s_3]|0.7\},\{[s_1,s_2]|0.8,[s_2,s_3]|0.2\},\{[s_3,s_4]|0.4,[s_4,s_5]|0.6\}\rangle\\ \beth_4: & \langle\{[s_1,s_2]|0.3,[s_2,s_3]|0.7\},\{[s_1,s_2]|0.5,[s_2,s_3]|0.5\},\{[s_3,s_4]|0.8,[s_4,s_5]|0.2\}\rangle\end{pmatrix}$$

Step 6. Combining the Euclidean distance formula (Formula (9)) with the weight vector w =

$(0.273, 0.430, 0.177, 0.120)^{\mathrm{T}}$, the preference values of each alternative are calculated using Formulas (21)-(22), as shown in Table 5.

Table 5 .Preference values of each alternative

	$D_H\left(\tilde{r}_{ij}(p), \tilde{Q}^{+}(p)\right)$					$D_H\left(\tilde{r}_{ij}(p), \tilde{Q}^{-}(p)\right)$				
	a_1	a_2	a_3	a_4	VD_i^{+}	a_1	a_2	a_3	a_4	VD_i^{+}
h_1	4.887	4.935	0.000	0.000	3.456	2.389	10.083	11.563	7.499	7.935
h_2	10.685	14.532	7.683	7.499	11.425	7.883	0.000	5.429	0.000	3.113
h_3	0.000	6.031	11.563	5.787	5.334	5.873	15.434	0.000	7.657	9.159
h_4	5.873	0.000	10.195	2.205	3.672	0.000	14.487	6.266	7.103	8.191

Step 7. The utility values are calculated using Formulas (23)-(24), and the normalized value for each alternative is obtained for ranking evaluation. The results are shown in Table 6.

Step 8. The ranking value of each alternative is determined using Formula (25). The results are shown in Table 6.

Table 6. Calculation results

	U_i^{+}	U_i^{-}	RA_i	*Ranking*
h_1	1.000	0.866	0.933	1
h_2	0.303	0.340	0.321	4
h_3	0.648	1.000	0.824	3
h_4	0.941	0.894	0.918	2

Step 9. From the results in Table 6, the ranking of the alternatives is $h_1 \succ h_4 \succ h_3 \succ h_2$.

4.2 Sensitivity analysis

To test the impact of changes in the parameter$q$ on the criteria weights and the ranking results of alternatives, under the condition that $0 \leq \left(max_{t=1}^{\#T}\mu_{\varsigma(t)}^{U}\right)^{q} + \left(max_{r=1}^{\#R}\eta_{\varsigma(t)}^{U}\right)^{q} + \left(max_{w=1}^{\#W}\nu_{\varsigma(w)}^{U}\right)^{q} \leq k^{q}$, it is calculated that $q \geqslant 2.6422$, and the smallest integer *q*=3 is taken. With the parameter$q$ taking values in the range$[3,7]$, the utility values of each alternative and their rankings are shown in Table 7 and Figure 2.

Table 7. Sensitivity analysis results for different values of *q*

q	3	3.5	4	4.5	5	5.5	6	6.5	7
h_1	0.933	0.966	0.972	0.977	0.982	0.986	0.990	0.994	0.998
h_2	0.321	0.175	0.154	0.134	0.118	0.102	0.088	0.075	0.064
h_3	0.824	0.813	0.801	0.792	0.784	0.777	0.770	0.762	0.754
h_4	0.918	0.954	0.939	0.926	0.912	0.901	0.888	0.873	0.860

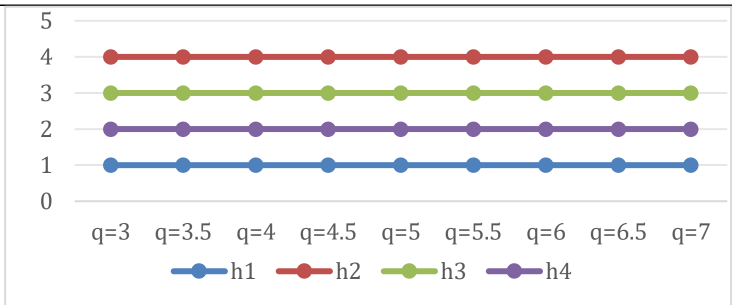

Fig 2. Ranking changes with different values of q

The sensitivity analysis results shown in Figure 2 indicate that when the value of q varies within the range of 3 to 7, the RA_i values of each alternative exhibit differentiated trends, but $h_2$always remains in the last place. From the perspective of the intrinsic mechanism of parameter influence, the subjective weights of the criteria calculated by the FUCOM method remain fixed and do not change with the value of q. However, the value of q directly affects the calculation process of the q-th power of the Euclidean distance in the ITARA method, thereby altering the objective weights and combined weights of the criteria. The specific variation patterns are shown in Table 8 and Figure 3: the weights of criteria w_1, w_3 and w_4 show a continuous downward trend, while the weight of the core environmental criterion w_2 significantly increases. It is worth noting that although the criteria weights undergo the above regular adjustments with the change of q, throughout the entire range of q values from 3 to 7, the ranking of alternatives remains stable as $h_1 \succ h_4 \succ h_3 \succ h_2$, without any reversal. This result fully verifies the strong robustness of the model constructed in this paper. At the same time, the adjustability of the q value endows the model with good scenario adaptability. Decision-makers can flexibly select an appropriate q value according to their actual risk preferences and uncertainty tolerance, achieving accurate screening of the optimal circular supplier.

Table 8. Criteria results for parameter *q*

q	w_1	w_2	w_3	w_4
3	0.273	0.430	0.177	0.120
3.5	0.263	0.444	0.180	0.113
4	0.253	0.461	0.182	0.104
4.5	0.243	0.480	0.182	0.095
5	0.232	0.500	0.182	0.086
5.5	0.221	0.521	0.180	0.078
6	0.210	0.542	0.179	0.069
6.5	0.198	0.562	0.177	0.063
7	0.187	0.583	0.174	0.056

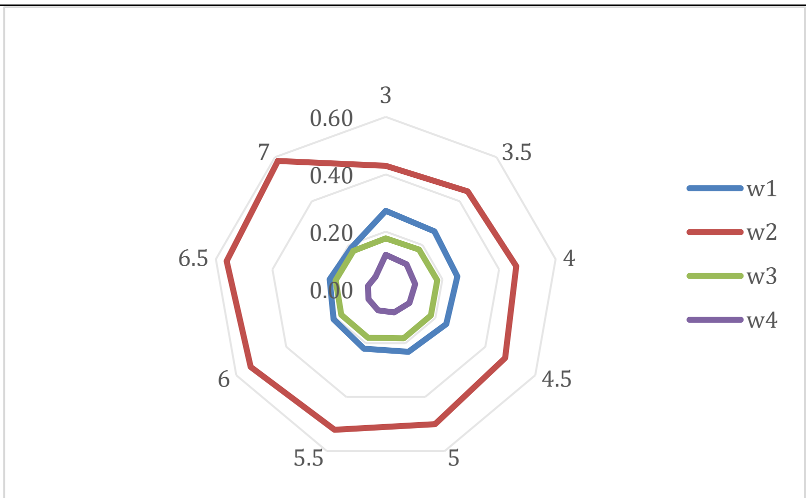

Fig 3. Weight changes with different values of *q*

4.3 Comparative analysis and discussion

To verify the rationality, effectiveness and superiority of the PULTSFS-FUCOM-ITARA-CRADIS multi-criteria decision-making method proposed in this paper, the classical TOPSIS and VIKOR methods are extended to the PULTSF environment, and the PULTSFWA and PULTSFWG multi-attribute information aggregation operators in the PULTSF environment are introduced as comparative methods[22][23]. All comparative methods use the same standardized PULTSF decision matrix and the combined weight vector $w = (0.273, 0.430, 0.177, 0.120)^T$ obtained from the FUCOM-ITARA combination weighting**.**

Table 9. Comparison results of different methods

Method name	Calculation results	Decision ranking	Optimal alternative
PULTSF-TOPSIS	U_1^-: 0.697, U_2^-: 0.214, U_3^-: 0.632, U_4^-: 0.690	$h_1 \succ h_4 \succ h_3 \succ h_2$	h_1
PULTSF-VIKOR	ρ=0.2 CV_1: 0.000, CV_2: 1.000, CV_3: 0.400, CV_4:0.277	$h_1 \succ h_4 \succ h_3 \succ h_2$	h_1
	ρ=0.5 CV_1: 0.000, CV_2: 1.000, CV_3: 0.354, CV_4:0.272	$h_1 \succ h_4 \succ h_3 \succ h_2$	h_1

	ρ=0.8 CV_1: 0.000, CV_2: 1.000, CV_3: 0.307, CV_4:0.268	$h_1 \succ h_4 \succ h_3 \succ h_2$	h_1
PULTSFWA	h_1: 0.537, h_2: 0.490, h_3 : 0.505, h_4: 0.563	$h_4 \succ h_1 \succ h_3 \succ h_2$	h_4
PULTSFWG	h_1: 0.466, h_2: 0.414, h_3 : 0.455, h_4: 0.519	$h_4 \succ h_1 \succ h_3 \succ h_2$	h_4
PULTSF-CRADIS	RA_1: 0.933, RA_2: 0.321, RA_3: 0.824, RA_4: 0.918	$h_1 \succ h_4 \succ h_3 \succ h_2$	h_1

As shown in Table 9 and Figure 4, the four methods—PULTSF-TOPSIS, PULTSF-VIKOR, PULTSFWA, and PULTSFWG—together with the PULTSFS-FUCOM-ITARA-CRADIS method proposed in this paper, are applied to the supplier selection problem in the context of the circular economy. Among them, PULTSF-TOPSIS and PULTSF-VIKOR yield exactly the same ranking results as the method proposed in this paper, all identifying alternative h_1 as the optimal solution, with the specific ranking being $h_1 \succ h_4 \succ h_3 \succ h_2$. In contrast, the ranking results of the PULTSFWA and PULTSFWG methods are consistent with each other but differ from the former; these two methods select h_4 as the optimal solution, with the ranking being $h_4 \succ h_1 \succ h_3 \succ h_2$. The differences in the above ranking results essentially stem from fundamental differences in the core decision-making mechanisms of each method under the PULTSF fuzzy decision environment.

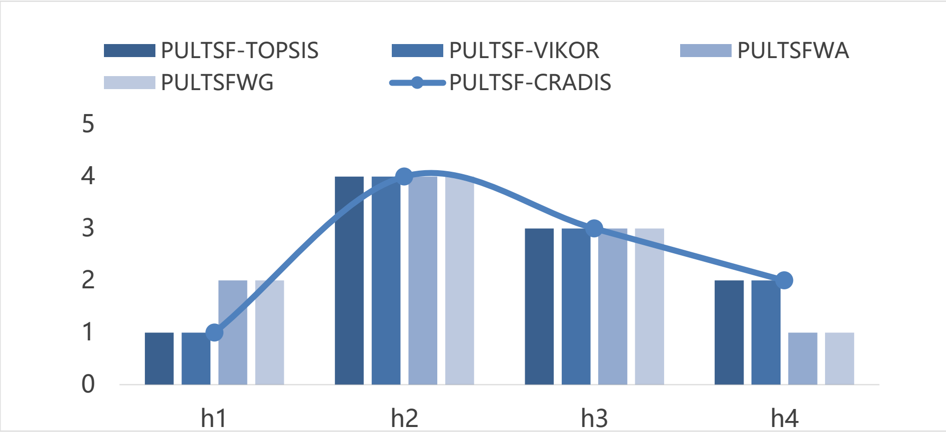

Fig .4 Ranking of alternatives by different comparative methods

The PULTSF-TOPSIS method calculates the weighted separation measures between alternatives and the PULTSF positive and negative ideal solutions, solves the relative closeness coefficient, and ranks alternatives accordingly. Its core lies in quantifying the degree of closeness between an alternative and the ideal solution through a single indicator. The PULTSF-VIKOR method calculates the group utility value and the individual regret value, and solves a compromise value that integrates these two indicators to complete the ranking, balancing the dual decision logic of maximizing group utility and minimizing individual regret. Both methods take the ideal solution as the core judgment benchmark, evaluating the comprehensive performance of alternatives by quantifying their deviation from the ideal solution. Their computational logic is intuitive and concise, sharing commonalities with the core decision logic of the method proposed in this paper, and thus yield consistent ranking results.

In contrast, the PULTSFWA and PULTSFWG methods adopt a linear aggregation approach for multi-attribute information. The PULTSFWA operator aggregates the PULTSFNs under each attribute through weighted arithmetic means, while the PULTSFWG operator completes information integration through weighted geometric means. Both methods quantify the aggregated comprehensive value using a score function and then rank the alternatives according to the score values. These operator-based methods can only achieve simple linear fusion of evaluation information. Unlike CRADIS-type methods, they cannot fully reflect the comprehensive compromise performance of alternatives under multiple criteria through dual deviation measurements from positive and negative ideal solutions, making it difficult to accurately adapt to the complex fuzzy decision-making scenarios of circular supplier selection. This is the core reason for the differences in ranking results compared with the method proposed in this paper.It is worth noting that the method proposed in this paper incorporates the combined weights obtained from FUCOM-ITARA combination weighting, and improves the ideal solution approximation logic of CRADIS based on the PULTSFS-specific score function and the proposed Euclidean distance. This process not only balances the subjective experience of decision-makers with the objective characteristics of the data through combination weighting, ensuring the reliability of attribute weights, but also accurately quantifies the deviation of alternatives from the ideal solution under the PULTSF fuzzy environment, fully capturing the comprehensive performance of alternatives under multiple criteria.

5. Conclusion

To address the issues in the complex multi-criteria decision-making problem of circular supplier selection—such as the fuzziness of evaluation information, neutral cognition, and probabilistic distribution characteristics, the insufficient adaptability of existing CRADIS methods in complex fuzzy environments, the lack of scientific rigor in weight determination, and the incomplete representation of information—this paper proposes a Euclidean distance measure based on the PULTSFS. It integrates the FUCOM subjective weighting method and extends and optimizes the ITARA weighting method in the PULTSF environment. Through combined subjective–objective weighting and an adapted CRADIS ranking model, a PULTSF-FUCOM-ITARA-CRADIS framework is constructed, providing a precise solution for supplier selection in the context of the circular economy. As the first study in this series, this paper establishes a complete foundational framework encompassing “fundamental theoretical innovation, methodological system construction, and empirical case validation.” The relevant research conclusions and implications are as follows:

First, the introduction of PULTSFS enables a comprehensive and precise representation of complex evaluation information, addressing the expressive limitations of traditional fuzzy sets. By integrating the interval linguistic terms and probabilistic quantification capabilities of PULTS with the membership–neutrality–non-membership three-dimensional structure of T-SFS, PULTSFS not only allows experts to express evaluation preferences using multiple linguistic terms combined with probability distributions, but also effectively captures neutral attitudes and confidence differences in decision-making. This overcomes the limitation of existing two-dimensional fuzzy sets in representing neutral information. Based on the concept of PULTSFS, this paper proposes a Euclidean distance measure and improves the basic operational rules of PULTSFN. Its generality allows it to degenerate into various special forms such as PULqROFS and PULIFS under specific conditions, adapting to circular supplier evaluation scenarios of varying complexity, thereby offering a more flexible tool for quantifying and distinguishing highly uncertain information.

Second, the combined weighting method of “FUCOM subjective weighting + improved ITARA objective weighting” constructed in this paper achieves the synergistic integration of subjective preferences and objective data, ensuring the scientific rigor and full consistency of weight allocation. The FUCOM method establishes a three-step framework of “criterion correlation analysis, pairwise comparison judgment, and full consistency testing.” In the case study, the resulting weight DFC value is 0, demonstrating that the weights are unbiased and fully satisfy the consistency requirement, effectively overcoming the excessive subjectivity inherent in traditional subjective weighting methods. At the same time, to address issues in the original ITARA method—such as the subjective determination of indifference thresholds and the distortion of fuzzy information during quantification—it is extended to the probabilistic uncertain linguistic T-spherical fuzzy environment. The indifference threshold is objectively calculated using the minimum Euclidean distance between pairs of alternatives under the same criterion, and the effective deviation is measured by combining the Euclidean distance between alternatives and the positive ideal solution. The resulting objective weights reflect the discriminative power of the criteria, balancing decision-maker preferences while fully exploiting objective information from the decision data, making the weight determination logic more scientific.

Third, the optimized CRADIS ranking model significantly enhances decision adaptability and robustness in complex fuzzy environments. Based on the proposed PULTSFS Euclidean distance, this paper improves the ideal solution approximation logic and ranking calculation process of CRADIS. It clearly distinguishes cost-type and benefit-type criteria and completes standardization conversion. By calculating the weighted distances between each alternative and the positive and negative ideal solutions, combined with utility values and ranking values, it achieves rational ranking of alternatives, effectively avoiding the ranking reversal problem common in traditional MCDM methods. This makes it more suitable for the multi-objective balancing decision-making demands of "economy-environment-society-circularity" in circular supplier evaluation.

Fourth, the case analysis, sensitivity analysis, and comparative verification of circular supplier selection fully demonstrate the practicality, robustness, and superiority of the proposed model. Taking the selection of circular suppliers for a new energy battery enterprise as a case study, sensitivity analysis shows that when the q value varies within the range of 3 to 7, the ranking of alternatives remains stable without any reversal, reflecting the strong robustness of the model. At the same time, the adjustability of the q value endows the model with flexible adaptability to different levels of uncertainty tolerance. Comparative analysis shows that the ranking results of the proposed model are completely consistent with those of the PULTSF-TOPSIS and PULTSF-VIKOR methods, verifying the rationality of the decision results. In contrast to linear aggregation methods such as PULTSFWA and PULTSFWG, the proposed model more comprehensively reflects the comprehensive compromise performance of alternatives through dual deviation measurements from the ideal solution and the anti-ideal solution, offering greater decision adaptability in complex and uncertain environments.

As the first paper in this series, this study has completed the construction of core theories and foundational methods. However, certain limitations remain, which also represent the core directions for subsequent research in this series: First, the model does not consider the interrelationships among the evaluation criteria for circular suppliers, whereas in practice, criteria such as circular capability, environmental performance, economic cost, and social contribution exhibit significant interactions. Second, the model is constructed based on the assumption of complete rationality and does not account for the influence of decision-makers' bounded rationality on decision outcomes. Third, the case analysis focuses on the new energy battery industry and still adopts a traditional evaluation system without incorporating ESG concepts to build a more universally applicable evaluation system for circular suppliers. The generalizability of the model also requires further validation.

To address these limitations, subsequent research in this series will be carried out in two stages: The second paper will build upon the basic methodological framework established in this paper, considering the interactive effects among the quantitative criteria, while integrating behavioral decision theories such as cumulative prospect theory, regret theory, or disappointment theory to accommodate the bounded rationality characteristics of decision-makers, thereby further enhancing the scientific rigor and practical adaptability of the methodological system. The third paper will, based on the extended model from the second paper, construct a multi-dimensional evaluation system for circular resilient suppliers centered on the ESG concept. It will focus on case analysis and conduct empirical research to deepen the practical application of the model, providing references for enterprises, governments, and society in the selection of circular resilient suppliers, and offering more systematic and practically operational decision support for sustainable supply chain management in the context of the circular economy.

Author Contributions

Conceptualization, H.W.; methodology, H.W. and K.Z.; formal analysis, H.W.; investigation, H.W. and K.Z.; resources, H.W.; writing-original draft preparation, H.W. and K.Z.; writing-review and editing,K.Z. and N.Z.; visualization, N.Z.; supervision, H.W.; project administration, H.W.; funding acquisition,H.W. All authors have read and agreed to the published version of the manuscript.

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Data Availability Statement

Not applicable.

Conflicts of Interest

The authors declare no conflict of interest.

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