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Multiple Attribute Decision Method Based on Neutrosophic Z-Number Frank Combined Compromise Solution Approach

Dongmei Li[1], Yuan Rong [2,*]

1 School of Economics and Management, Shanghai Zhongqiao Vocational and Technical University, Shanghai, China
2 School of Medical Information and Engineering, Ningxia Medical University, Yinchuan, China

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ABSTRACT

Uncertain multi-attribute decision-making (MADM) represents a fundamental area within decision science, with extensive applications across various fields. The neutrosophic Z-number (NZN) offers a powerful framework for modeling uncertain information, as it captures not only the imprecise, incomplete, and inconsistent preferences of decision-makers but also the associated reliability of those preferences. This study develops a novel MADM approach by integrating the Combined Compromise Solution (CoCoSo) method, Frank aggregation operators, and the Logarithmic Decomposition of Criteria Importance (LODECI) model within the NZN context. Firstly, operational rules for NZNs are defined based on Frank triangular norms. Subsequently, the neutrosophic Z-number Frank weighted averaging (NZNFWA) and neutrosophic Z-number Frank weighted geometric (NZNFWG) operators, along with their ordered variants, are proposed to enhance the flexibility and robustness of aggregating NZN evaluations. To objectively determine attribute weights, an extended LODECI model grounded in the NZN score function is introduced. The enhanced CoCoSo method is then applied, utilizing the proposed aggregation operators to derive the final ranking of alternatives. The applicability and effectiveness of the proposed framework are demonstrated through a case study assessing green logistics development levels. Furthermore, a parameter sensitivity analysis and a comparative analysis with existing NZN-based methods are conducted to validate its robustness and superiority. The results confirm that the proposed NZN-LODECI-CoCoSo approach exhibits strong stability and practicality, providing an effective solution for complex decision-making problems under uncertainty.

1. Introduction

Multi-attribute decision-making, as a vital branch of decision science, fundamentally aims to identify the optimal alternative from a predetermined set of options based on a defined set of attributes using MADM theories and methodologies, and has been extensively applied across numerous fields. With the growing complexity of decision environments and the increasing uncertainty inherent in decision-makers' cognitive capabilities, MADM theories and methods

*Corresponding author: Yuan Rong
E-mail address: rongyuanry@163.com

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grounded in uncertain preference information have garnered significant scholarly attention. However, real-world decision scenarios are often characterized by fuzziness, incompleteness, and the unreliability of information sources, rendering the accurate characterization of the true semantics of decision information a critical challenge in this domain.

To represent the uncertainty in preference information arising from the cognitive fuzziness of decision-makers, Zadeh introduced fuzzy set theory based on membership functions to address real-world uncertainties [1]. Subsequently, to enhance the expressive power of fuzzy set theory in capturing uncertain information, extensions such as intuitionistic fuzzy sets [2] and interval-valued intuitionistic fuzzy sets [3] were proposed, which depict fuzzy information by simultaneously considering membership, non-membership, and hesitation degrees. Nevertheless, these fuzzy sets and their variants struggle to effectively represent inconsistent and incomplete information. In response, Smarandache [4] proposed neutrosophic set theory from a philosophical perspective, which characterizes uncertain, inconsistent, and discontinuous information through truth-membership, indeterminacy-membership, and falsity-membership functions. Given that neutrosophic sets are defined on non-standard subintervals, their direct application in engineering sciences proved challenging. To address this, Wang [5] introduced single-valued neutrosophic sets, utilizing truth, falsity, and indeterminacy membership functions to collectively represent decision information. Since its inception, neutrosophic set theory has been extensively studied with numerous scholars exploring its extensions, theoretical foundation and applications in MADM yielding substantial research outcomes [6, 7, 8, 9]. Despite the notable advantages of neutrosophic sets in modeling uncertainty, they fail to adequately account for the reliability of decision-makers' preference information.

To address the issue of evaluating information reliability, Zadeh proposed Z-number theory based on fuzzy set theory, which employs an ordered pair to simultaneously represent the fuzziness of information and the reliability measure associated with that fuzziness. Leveraging the advantages of Z-numbers in representing uncertain information, Z-number theory has been widely investigated and integrated with various fuzzy set theories, resulting in a series of extended Z-number formulations for handling complex uncertain information [10, 11, 12]. Combining the strengths of Z-numbers and neutrosophic sets, Du et al. [13] introduced neutrosophic Z-number theory and defined the neutrosophic Z-number weighted averaging and weighted geometric aggregation operators. The core concept involves employing the Z-number structure to characterize truth, falsity, and indeterminacy membership functions, thereby providing both a representation of uncertain information and its associated reliability. This approach enhances the authenticity of decision preference information and improves the scientific rigor and rationality of decision outcomes. Further, Ye [14] proposed the generalized distance and similarity measures between NZN sets, and introduced cosine and cotangent similarity measures. Ye et al. [15] defined the NZN Aczel-Alsina operations and the proposed a novel MADM based on some novel NZN Aczel-Alsina weighted aggregation operators. Ye et al. [16] presented a MADM approach on the basis of NZN Dombi aggregation operators. Karabacak [17] proposed new correlation coefficient on NZN set and used them to resolve medical diagnostic problem. Kamran et al. [18] introduced the notion of NZN rough set and constructed a sine trigonometric operators-based MADM approach for the assessment of sustainable industry. In order to determine the priority among the fused NZNs, Wu et al. [19] proposed a novel score function for comparing NZNs and presented some NZN Schweizer-Sklar prioritized aggregation operators to build MADM method. Abbas et al. [20] proposed a hybrid decision method with NZN information for smart solar panel evaluation. The mentioned research outcomes confirm that NZN is a powerful uncertain information representation model for tackling complex decision problems. However, the Frank operators-based NZN decision method has not been investigated in previous references.

Decision model is a vital part during the process of decision analysis. In the past years, different theories-based on decision approaches are developed for unfold decision analysis and assessment applications, such as COPRAS [21], ARAS [22], VIKOR [23] and so on. The CoCoSo decision model is a powerful MADM approach through merging the weighted sum model, weighted product model and three fusion strategies [24, 25]. Based on the analysis of the structure of CoCoSo method, we can conclude that it has the following distinct merits: (1) tit evaluates the performance of solutions from multiple perspectives by integrating weighted sum and weighted product models, effectively avoiding bias in a single evaluation logic and significantly improving the reliability and robustness of decision results; (2) it can balance potential conflicts between various evaluation criteria and select the compromise solution with the best comprehensive performance; (3) it has clear computational logic and can be easily combined with various weighting methods, demonstrating wide applicability. Based on the mentioned advantages, many extended CoCoSo approach based on different fuzzy set theory are propounded to construct decision analysis framework [26, 27, 28, 29]. Besides, Pamucar and Gorcun [30] presented the hybrid LBWA-CoCoSo'B techniques to assess the European container ports, where fuzzy LBWA is used for estimate the subjective weight of criteria. Deveci et al. [31] built new MCDM model called FUCOM-CoCoSo based on Hamacher operator for selecting he appropriate floating offshore wind farm. Zhang and Wei [32] reported the an improved of CoCoSo method based on cumulative prospect theory with spherical fuzzy information to ponder the risk preference of expert. Wang et al. [33] propounded the novel CoCoSo method for group MCDM problem based on Frank softmax operator under T-spherical fuzzy setting. However, the CoCoSo method has not yet been extended to build decision methods in the NZN setting.

Although the neutrosophic Z-number (NZN) has proven effective in capturing uncertain, incomplete, and inconsistent information along with its reliability. The CoCoSo method is demonstrated as a robustness and stability approach for alternative ranking. Decision methods based on NZN information in complex decision environments still faces the following motivations:

- Existing aggregation operators under NZN context lack the flexibility to accommodate diverse decision-maker attitudes;
- Current methods for determining attribute weights fail to fully leverage the reliability information embedded in NZNs;
- There is a lack of robust decision frameworks that integrate the advantages of NZNs with advanced compromise ranking models.

Therefore, this study is motivated to develop a comprehensive NZN-CoCoSo method that enhances information fusion flexibility, improves objectivity in weight determination, and ensures stability in alternative prioritization under uncertainty. The contributions of this study are described as follows:

✓ We innovatively define operational laws for NZNs based on Frank T-norm and S-norm. Then the NZNFWA, NZNFWG operators and their ordered weighted averaging operator are proposed. This significantly improves the flexibility and adaptability of aggregating NZN assessment information by incorporating adjustable parameters.

✓ We propose an enhanced LODECI model utilizing the NZN score function to determine the objective weight of attribute. This extension enables a more reliable determination of attribute weights by effectively capturing both the truth and reliability dimensions of NZNs.

✓ We construct a novel NZN-LODECI-CoCoSo decision framework by embedding the proposed Frank aggregation operators into the CoCoSo method. This integration ensures robust compromise ranking of alternatives that fully respects the features of neutrosophic Z-number information.

- ✓ We validate the practicality, high stability, and superiority of the proposed method over existing NZN-based approaches in handling complex and uncertain MADM problems by a case study on green logistics development evaluation, parameter analysis and comparative discussions.

The remainder of this paper is organized as follows. Section 2 recalls fundamental concepts of NZN and Frank norms. In Section 3, we define operational rules for NZNs based on Frank norms and propose several novel NZN Frank aggregation operators. Section 4 elaborates on the NZN-LODECI-CoCoSo decision framework and delineates its detailed decision steps. Section 5 presents a case study on green logistics development evaluation, accompanied by parameter sensitivity analysis and comparative discussions. Finally, Section 6 concludes the study with a conclusion.

2. Basic knowledge

This section recalls basic conceptions of NZN and Frank t-norms, which serve as the foundation for the construction of decision approach.

Neutrosophic set is proposed to model the uncertain and incomplete information from the perspective of truth-membership, indeterminacy-membership and falsity-membership grades. However, Neutrosophic set fails to reflect the reliability of every part of the preference information, thus NZN is proposed by combining the Neutrosophic set and Z-number to characterize the uncertainty and reliability of information Simultaneously[13].

Definition 1[13]. Let Y be a given universe of discourse. Then a neutrosophic Z-number subset on the universe is expressed as:

$$\overline{NZ}=\left\{\left\langle y,\left(\overline{TV}(y),\overline{T\Re}(y)\right),\left(\overline{IV}(y),\overline{I\Re}(y)\right),\left(\overline{FV}(y),\overline{F\Re}(y)\right)\right\rangle \middle| y\in Y\right\}, \quad (1)$$

where $\left(\overline{TV}(y),\overline{T\Re}(y)\right)$, $\left(\overline{IV}(y),\overline{I\Re}(y)\right)$ and $\left(\overline{FV}(y),\overline{F\Re}(y)\right)$ represent the truth-membership Z-number, indeterminacy-membership Z-number, and falsity-membership Z-number, respectively, and $Y\rightarrow[0,1]^2$, the initial part $\overline{V}$ of the three pairs stands for a neutrosophic values on Y, while the second part $\overline{\Re}$ denotes a neutrosophic reliability for $\overline{V}$. Neutrosophic Z-number meets the following conditions $0\le\overline{TV}(y)+\overline{IV}(y)+\overline{FV}(y)\le 3$ and $0\le\overline{T\Re}(y)+\overline{I\Re}(y)+\overline{F\Re}(y)\le 3$. For convenience, Neutrosophic Z-number (NZN) can be represented as $\overline{NZ}=\left\{\left(\overline{TV},\overline{T\Re}\right),\left(\overline{IV},\overline{I\Re}\right),\left(\overline{FV},\overline{F\Re}\right)\right\}$.

Definition 2[13]. Let $\overline{NZ}_j=\left\{\left(\overline{TV}_j,\overline{T\Re}_j\right),\left(\overline{IV}_j,\overline{I\Re}_j\right),\left(\overline{FV}_j,\overline{F\Re}_j\right)\right\}(j=1,2)$ be two NZNs. Then

(1) $\overline{NZ}_1\subseteq\overline{NZ}_2\Rightarrow\overline{TV}_1\le\overline{TV}_2,\overline{T\Re}_1\le\overline{T\Re}_2,\overline{IV}_1\le\overline{IV}_2,\overline{I\Re}_1\le\overline{I\Re}_2,\overline{FV}_1\ge\overline{FV}_2,\overline{F\Re}_1\ge\overline{F\Re}_2$;

(2) $\overline{NZ}_1=\overline{NZ}_2\Leftrightarrow\overline{NZ}_1\subseteq\overline{NZ}_2,\overline{NZ}_1\supseteq\overline{NZ}_2$;

(3) $\overline{NZ}_1\oplus\overline{NZ}_2=\left\{\begin{array}{l}\left(\overline{TV}_1+\overline{TV}_2-\overline{TV}_1\overline{TV}_2,\overline{T\Re}_1+\overline{T\Re}_2-\overline{T\Re}_1\overline{T\Re}_2\right),\\ \left(\overline{IV}_1\overline{IV}_2,\overline{I\Re}_1\overline{I\Re}_2\right),\left(\overline{FV}_1\overline{FV}_2,\overline{F\Re}_1\overline{F\Re}_2\right)\end{array}\right\}$;

(4) $\overline{NZ}_1 \otimes \overline{NZ}_2 = \left\{ \begin{array}{l} \left(\overline{TV}_1 \overline{TV}_2, \overline{T\Re}_1 \overline{T\Re}_2 \right), \left(\overline{IV}_1 + \overline{IV}_2 + \overline{IV}_1 \overline{IV}_2, \overline{I\Re}_1 + \overline{I\Re}_2 + \overline{I\Re}_1 \overline{I\Re}_2 \right), \\ \left(\overline{FV}_1 + \overline{FV}_2 + \overline{FV}_1 \overline{FV}_2, \overline{F\Re}_1 + \overline{F\Re}_2 + \overline{F\Re}_1 \overline{F\Re}_2 \right) \end{array} \right\}$

(5) $\left(\overline{NZ}_1\right)^c = \left\{ \left(\overline{FV}_1, \overline{F\Re}_1\right), \left(1-\overline{IV}_1, 1-\overline{I\Re}_1\right), \left(\overline{TV}_1, \overline{T\Re}_1\right) \right\}$;

(6) $\lambda \overline{NZ}_1 = \left\{ \left(1-\left(1-\overline{TV}_1\right)^{\lambda}, 1-\left(1-\overline{T\Re}_1\right)^{\lambda}\right), \left(\left(\overline{IV}_1\right)^{\lambda}, \left(\overline{I\Re}_1\right)^{\lambda}\right), \left(\left(\overline{FV}_1\right)^{\lambda}, \left(\overline{F\Re}_1\right)^{\lambda}\right) \right\}$;

(7) $\left(\overline{NZ}_1\right)^{\lambda} = \left\{ \left(\left(\overline{TV}_1\right)^{\lambda}, \left(\overline{T\Re}_1\right)^{\lambda}\right), \left(1-\left(1-\overline{IV}_1\right)^{\lambda}, 1-\left(1-\overline{I\Re}_1\right)^{\lambda}\right), \left(1-\left(1-\overline{FV}_1\right)^{\lambda}, 1-\left(1-\overline{F\Re}_1\right)^{\lambda}\right) \right\}$.

Definition 3[13]. Assume that $\overline{NZ}_j = \left\{ \left(\overline{TV}_j, \overline{T\Re}_j\right), \left(\overline{IV}_j, \overline{I\Re}_j\right), \left(\overline{FV}_j, \overline{F\Re}_j\right) \right\} \left(j = 1(1)n\right)$ be a set of NZNs. The its score function is defined as

$$SF\left(\overline{NZ}_j\right) = \frac{1}{3}\left(2 + \overline{TV}_j \overline{T\Re}_j - \overline{IV}_j \overline{I\Re}_j - \overline{FV}_j \overline{F\Re}_j\right), SF\left(\overline{NZ}_j\right) \in [0,1] \quad (2)$$

Wu et al.[19] discussed the shortcoming of score function is Definition 3, and then proposed a new score function.

Definition 4[19]. Let $\overline{NZ}_j = \left\{ \left(\overline{TV}_j, \overline{T\Re}_j\right), \left(\overline{IV}_j, \overline{I\Re}_j\right), \left(\overline{FV}_j, \overline{F\Re}_j\right) \right\} \left(j = 1(1)n\right)$ be a set of NZNs. The its score function is defined as

$$SF\left(\overline{NZ}_j\right) = \frac{1}{6}\left(\begin{array}{l} 4 + \left(\overline{TV}_j\right)^2 \left(\overline{T\Re}_j\right)^2 - \left(\overline{IV}_j\right)^2 \left(\overline{I\Re}_j\right)^2 - \left(\overline{FV}_j\right)^2 \left(\overline{F\Re}_j\right)^2 \\ + \overline{TV}_j \overline{T\Re}_j - \overline{IV}_j \overline{I\Re}_j - \overline{FV}_j \overline{F\Re}_j \end{array} \right) \quad (3)$$

wherein $SF\left(\overline{NZ}_j\right) \in [0,1]$. For any two NZNs $\overline{NZ}_1$ and $\overline{NZ}_2$, if $SF\left(\overline{NZ}_1\right) > SF\left(\overline{NZ}_2\right)$,则 $\overline{NZ}_1 \succ \overline{NZ}_2$; if $\Theta\left(\overline{NZ}_1\right) < \Theta\left(\overline{NZ}_2\right)$, then $\overline{NZ}_1 \prec \overline{NZ}_2$; if $\Theta\left(\overline{NZ}_1\right) = \Theta\left(\overline{NZ}_2\right)$, then $\overline{NZ}_1 \square \overline{NZ}_2$.

Definition 5[13]. Let $\overline{NZ}_j = \left\{ \left(\overline{TV}_j, \overline{T\Re}_j\right), \left(\overline{IV}_j, \overline{I\Re}_j\right), \left(\overline{FV}_j, \overline{F\Re}_j\right) \right\} \left(j = 1(1)n\right)$ be a set of NZNs, ϑ_j is the weight of NZN $\overline{NZ}_j$ meeting $\sum_{j=1}^{n} \vartheta_j = 1,\ \vartheta_j \in [0, 1]$. Then Neutrosophic Z-number weighted averaging（NZNWA）operator and Neutrosophic Z-number weighted geometric（NZNWG）operator are defined as

$$NZWA\left(\overline{NZ}_1, \overline{NZ}_2, \cdots, \overline{NZ}_n\right) = \vartheta_1 \overline{NZ}_1 \oplus \vartheta_2 \overline{NZ}_2 \oplus \cdots \oplus \vartheta_n \overline{NZ}_n$$
$$= \left\{ \begin{array}{l} \left(1 - \prod_{j=1}^{n}\left(1-\overline{TV}_j\right)^{\vartheta_j}, 1 - \prod_{j=1}^{n}\left(1-\overline{T\Re}_j\right)^{\vartheta_j}\right), \left(\prod_{j=1}^{n}\left(\overline{IV}_j\right)^{\vartheta_j}, \prod_{j=1}^{n}\left(\overline{I\Re}_j\right)^{\vartheta_j}\right), \\ \left(\prod_{j=1}^{n}\left(\overline{FV}_j\right)^{\vartheta_j}, \prod_{j=1}^{n}\left(\overline{F\Re}_j\right)^{\vartheta_j}\right) \end{array} \right\} \quad (4)$$

$$NZWG\left(\overline{NZ}_1,\overline{NZ}_2,\cdots,\overline{NZ}_n\right)=\left(\overline{NZ}_1\right)^{\vartheta_1}\otimes\left(\overline{NZ}_2\right)^{\vartheta_2}\otimes\cdots\otimes\left(\overline{NZ}_n\right)^{\vartheta_n}$$
$$=\left\{\begin{array}{l}\left(\prod_{j=1}^{n}\left(\overline{TV}_j\right)^{\vartheta_j},\prod_{j=1}^{n}\left(\overline{T\Re}_j\right)^{\vartheta_j}\right),\left(1-\prod_{j=1}^{n}\left(1-\overline{TV}_j\right)^{\vartheta_j},1-\prod_{j=1}^{n}\left(1-\overline{T\Re}_j\right)^{\vartheta_j}\right),\\ \left(1-\prod_{j=1}^{n}\left(1-\overline{FV}_j\right)^{\vartheta_j},1-\prod_{j=1}^{n}\left(1-\overline{F\Re}_j\right)^{\vartheta_j}\right)\end{array}\right\} \tag{5}$$

The Frank T-norm and S-norm is a special case of Archimedean norm and has been utilized to define the operations and aggregation operators in fuzzy set theory[34, 35, 36]. From the mentioned researches, we find that the Frank norms have the following advantages. (1) It can reflect the risk preference of decision experts by adjusting the parameter in Frank norms. (2) It can construct a fuzzy preference aggregation model with good transitivity and decomposability, avoiding information loss caused by improper operator selection.

Definition 6[37]. Let $a,\hat{b}\in[0,1]$ be two real numbers. Then Frank t-norm and Frank s-norm are stated as

$$T_F\left(a,\hat{b}\right)=a\otimes_F\hat{b}=\log_\Theta\left(1+\frac{\left(\Theta^a-1\right)\left(\Theta^{\hat{b}}-1\right)}{\Theta-1}\right), \tag{6}$$

$$S_F\left(a,\hat{b}\right)=a\oplus_F\hat{b}=1-\log_\Theta\left(1+\frac{\left(\Theta^{1-a}-1\right)\left(\Theta^{1-\hat{b}}-1\right)}{\Theta-1}\right). \tag{7}$$

where $\Theta\in(1,\ +\infty)$. It is obviously that Frank t-norm and Frank s-norm possesses two special cases: (1) If $\Theta\to1$, the Frank t-norm and Frank s-norms are degenerated into the Algebraic operations, namely, $T\left(a,\hat{b}\right)=a\hat{b}$ and $S\left(a,\hat{b}\right)=a+\hat{b}-a\hat{b}$. (2) 1f $\Theta\to\infty$, the Frank product and Frank sum are reduced to the Lukasiewicz operations, namely $T\left(a,\hat{b}\right)\to\max\left(0,a+\hat{b}+1\right)$ and $S\left(a,\hat{b}\right)\to\min\left(a+\hat{b},1\right)$.

3. New neutrosophic Z-number Frank operator

In this section, we first define Frank operations for NZNs based on the Frank t-norm and t-conorm. Then we construct several Frank aggregation operators and investigate their properties.

3.1 New neutrosophic Z-number Frank operations

Definition 7. Let $\overline{NZ}_j=\left\{\left(\overline{TV}_j,\overline{T\Re}_j\right),\left(\overline{IV}_j,\overline{I\Re}_j\right),\left(\overline{FV}_j,\overline{F\Re}_j\right)\right\}(j=1,2)$ be two NZNs and a real number δ, then the Neutrosophic Z-number Frank operations are defined as:

$$\overline{NZ}_1 \oplus_F \overline{NZ}_2 = \left\{ \begin{array}{l} \left(1-\log_\Theta\left(1+\frac{\left(\Theta^{1-\overline{TV}_1}-1\right)\left(\Theta^{1-\overline{TV}_2}-1\right)}{\Theta-1}\right), 1-\log_\Theta\left(1+\frac{\left(\Theta^{1-\overline{T\mathfrak{R}}_1}-1\right)\left(\Theta^{1-\overline{T\mathfrak{R}}_2}-1\right)}{\Theta-1}\right) \right) \\ \left(\log_\Theta\left(1+\frac{\left(\Theta^{\overline{IV}_1}-1\right)\left(\Theta^{\overline{IV}_2}-1\right)}{\Theta-1}\right), \log_\Theta\left(1+\frac{\left(\Theta^{\overline{I\mathfrak{R}}_1}-1\right)\left(\Theta^{\overline{I\mathfrak{R}}_2}-1\right)}{\Theta-1}\right) \right) \\ \left(\log_\Theta\left(1+\frac{\left(\Theta^{\overline{FV}_1}-1\right)\left(\Theta^{\overline{FV}_2}-1\right)}{\Theta-1}\right), \log_\Theta\left(1+\frac{\left(\Theta^{\overline{F\mathfrak{R}}_1}-1\right)\left(\Theta^{\overline{F\mathfrak{R}}_2}-1\right)}{\Theta-1}\right) \right) \end{array} \right\};$$

$$\overline{NZ}_1 \otimes_F \overline{NZ}_2 = \left\{ \begin{array}{l} \left(\log_\Theta\left(1+\frac{\left(\Theta^{\overline{TV}_1}-1\right)\left(\Theta^{\overline{TV}_2}-1\right)}{\Theta-1}\right), \log_\Theta\left(1+\frac{\left(\Theta^{\overline{T\mathfrak{R}}_1}-1\right)\left(\Theta^{\overline{T\mathfrak{R}}_2}-1\right)}{\Theta-1}\right) \right), \\ \left(1-\log_\Theta\left(1+\frac{\left(\Theta^{1-\overline{IV}_1}-1\right)\left(\Theta^{1-\overline{IV}_2}-1\right)}{\Theta-1}\right), 1-\log_\Theta\left(1+\frac{\left(\Theta^{1-\overline{I\mathfrak{R}}_1}-1\right)\left(\Theta^{1-\overline{I\mathfrak{R}}_2}-1\right)}{\Theta-1}\right) \right), \\ \left(1-\log_\Theta\left(1+\frac{\left(\Theta^{1-\overline{FV}_1}-1\right)\left(\Theta^{1-\overline{FV}_2}-1\right)}{\Theta-1}\right), 1-\log_\Theta\left(1+\frac{\left(\Theta^{1-\overline{F\mathfrak{R}}_1}-1\right)\left(\Theta^{1-\overline{F\mathfrak{R}}_2}-1\right)}{\Theta-1}\right) \right) \end{array} \right\};$$

$$\delta \odot_F \overline{NZ}_1 = \left\{ \left(\begin{array}{l} 1-\log_\Theta\left(1+\frac{\left(\Theta^{1-\overline{TV}_1}-1\right)^\delta}{(\Theta-1)^{\delta-1}}\right), \\ 1-\log_\Theta\left(1+\frac{\left(\Theta^{1-\overline{T\mathfrak{R}}_1}-1\right)^\delta}{(\Theta-1)^{\delta-1}}\right) \end{array} \right), \left(\begin{array}{l} \log_\Theta\left(1+\frac{\left(\Theta^{\overline{IV}_1}-1\right)^\delta}{(\Theta-1)^{\delta-1}}\right), \\ \log_\Theta\left(1+\frac{\left(\Theta^{\overline{I\mathfrak{R}}_1}-1\right)^\delta}{(\Theta-1)^{\delta-1}}\right) \end{array} \right), \left(\begin{array}{l} \log_\Theta\left(1+\frac{\left(\Theta^{\overline{FV}_1}-1\right)^\delta}{(\Theta-1)^{\delta-1}}\right), \\ \log_\Theta\left(1+\frac{\left(\Theta^{\overline{F\mathfrak{R}}_1}-1\right)^\delta}{(\Theta-1)^{\delta-1}}\right) \end{array} \right) \right\};$$

$$\left(\overline{NZ}_1\right)^{\wedge_F \delta} = \left\{ \left(\begin{array}{l} \log_\Theta\left(1+\frac{\left(\Theta^{\overline{TV}_1}-1\right)^\delta}{(\Theta-1)^{\delta-1}}\right), \\ \log_\Theta\left(1+\frac{\left(\Theta^{\overline{T\mathfrak{R}}_1}-1\right)^\delta}{(\Theta-1)^{\delta-1}}\right) \end{array} \right), \left(\begin{array}{l} 1-\log_\Theta\left(1+\frac{\left(\Theta^{1-\overline{IV}_1}-1\right)^\delta}{(\Theta-1)^{\delta-1}}\right), \\ 1-\log_\Theta\left(1+\frac{\left(\Theta^{1-\overline{I\mathfrak{R}}_1}-1\right)^\delta}{(\Theta-1)^{\delta-1}}\right) \end{array} \right), \left(\begin{array}{l} 1-\log_\Theta\left(1+\frac{\left(\Theta^{1-\overline{FV}_1}-1\right)^\delta}{(\Theta-1)^{\delta-1}}\right), \\ 1-\log_\Theta\left(1+\frac{\left(\Theta^{1-\overline{F\mathfrak{R}}_1}-1\right)^\delta}{(\Theta-1)^{\delta-1}}\right) \end{array} \right) \right\}.$$

Theorem 1. Let $\overline{NZ}_j = \left\{\left(\overline{TV}_j, \overline{T\mathfrak{R}}_j\right), \left(\overline{IV}_j, \overline{I\mathfrak{R}}_j\right), \left(\overline{FV}_j, \overline{F\mathfrak{R}}_j\right)\right\}\left(j=1(1)n\right)$ be a set of NZNs and $\delta, \delta_1, \delta_2 \geq 0.$ Then the following properties can be obtained:

(1) $\overline{NZ}_1 \oplus_F \overline{NZ}_2 = \overline{NZ}_2 \oplus_F \overline{NZ}_1.$

(2) $\overline{NZ}_1 \otimes_F \overline{NZ}_2 = \overline{NZ}_2 \otimes_F \overline{NZ}_1.$

(3) $\delta \odot_F \left(\overline{NZ}_1 \oplus_F \overline{NZ}_2\right) = \left(\delta \odot_F \overline{NZ}_1\right) \oplus_F \left(\delta \odot_F \overline{NZ}_2\right).$

(4) $\delta_1 \Box_F \overline{NZ}_1 \oplus \delta_2 \Box_F \overline{NZ}_1 = \left(\delta_1 + \delta_2\right) \Box_F \overline{NZ}_1$.

(5) $\left(\overline{NZ}_1 \otimes_F \overline{NZ}_2\right)^{\wedge_F \lambda} = \left(\overline{NZ}_1\right)^{\wedge_F \delta} \otimes_F \left(\overline{NZ}_2\right)^{\wedge_F \delta}$.

(6) $\left(\overline{NZ}_1\right)^{\wedge_F \delta_1} \otimes_F \left(\overline{NZ}_1\right)^{\wedge_F \delta_2} = \left(\overline{NZ}_1\right)^{\wedge_F \left(\delta_1 + \delta_2\right)}$.

Proof. Straightforward.

3.2 Neutrosophic Z-number Frank aggregation operators

The current section develops NZN Frank weighted and ordered weighted operators utilizing the Frank norms.

Definition 8. Let $\overline{NZ}_j = \left\{\left(\overline{TV}_j, \overline{T\Re}_j\right), \left(\overline{IV}_j, \overline{I\Re}_j\right), \left(\overline{FV}_j, \overline{F\Re}_j\right)\right\} \left(j = 1(1)n\right)$ be a set of NZNs, ϑ_j is the weight of NZN $\overline{NZ}_j$ meeting $\sum_{j=1}^{n} \vartheta_j = 1,\ \vartheta_j \in [0, 1]$. Then Neutrosophic Z-number Frank weighted averaging（NZNFWA）operator is depicted as:

$$NZNFWA\left(\overline{NZ}_1, \overline{NZ}_2, \ldots, \overline{NZ}_n\right) = \vartheta_1 \Box_F \overline{NZ}_1 \oplus_F \vartheta_2 \Box_F \overline{NZ}_2 \oplus_F \cdots \oplus_F \vartheta_n \Box_F \overline{NZ}_n. \quad (8)$$

Theorem 2. Let $\overline{NZ}_j = \left\{\left(\overline{TV}_j, \overline{T\Re}_j\right), \left(\overline{IV}_j, \overline{I\Re}_j\right), \left(\overline{FV}_j, \overline{F\Re}_j\right)\right\} \left(j = 1(1)n\right)$ be a set of SCFNs. Then the integration outcome of NZNs by NZNFWA operator is a NZN and represented as:

$$NZNFWA\left(\overline{NZ}_1, \overline{NZ}_2, \ldots, \overline{NZ}_n\right) =$$

$$\left\{\begin{pmatrix} 1 - \log_\Upsilon\left(1 + \prod_{s=1}^{n}\left(\Upsilon^{1-\overline{TV}_j} - 1\right)^{\vartheta_j}\right), \\ 1 - \log_\Upsilon\left(1 + \prod_{s=1}^{n}\left(\Upsilon^{1-\overline{T\Re}_j} - 1\right)^{\vartheta_j}\right) \end{pmatrix}, \begin{pmatrix} \log_\Upsilon\left(1 + \prod_{s=1}^{n}\left(\Upsilon^{\overline{IV}_j} - 1\right)^{\vartheta_j}\right), \\ \log_\Upsilon\left(1 + \prod_{s=1}^{n}\left(\Upsilon^{\overline{I\Re}_j} - 1\right)^{\vartheta_j}\right) \end{pmatrix}, \begin{pmatrix} \log_\Upsilon\left(1 + \prod_{s=1}^{n}\left(\Upsilon^{\overline{FV}_j} - 1\right)^{\vartheta_j}\right), \\ \log_\Upsilon\left(1 + \prod_{s=1}^{n}\left(\Upsilon^{\overline{F\Re}_j} - 1\right)^{\vartheta_j}\right) \end{pmatrix}\right\} \quad (9)$$

Proof. This theorem can be proved by mathematical induction. When $n = 2$, one has

$$NZNFWA\left(\overline{NZ}_1,\overline{NZ}_2\right)=\vartheta_1\Box_F\overline{NZ}_1\oplus_F\vartheta_2\Box_F\overline{NZ}_2$$

$$=\left\{\begin{pmatrix}1-\log_\Theta\left(1+\dfrac{\left(\Theta^{1-\overline{TV}_1}-1\right)^{\vartheta_1}}{(\Theta-1)^{\vartheta_1-1}}\right),\\1-\log_\Theta\left(1+\dfrac{\left(\Theta^{1-\overline{T\Re}_1}-1\right)^{\vartheta_1}}{(\Theta-1)^{\vartheta_1-1}}\right)\end{pmatrix},\begin{pmatrix}\log_\Theta\left(1+\dfrac{\left(\Theta^{\overline{IV}_1}-1\right)^{\vartheta_1}}{(\Theta-1)^{\vartheta_1-1}}\right),\\\log_\Theta\left(1+\dfrac{\left(\Theta^{\overline{I\Re}_1}-1\right)^{\vartheta_1}}{(\Theta-1)^{\vartheta_1-1}}\right)\end{pmatrix},\begin{pmatrix}\log_\Theta\left(1+\dfrac{\left(\Theta^{\overline{FV}_1}-1\right)^{\vartheta_1}}{(\Theta-1)^{\vartheta_1-1}}\right),\\\log_\Theta\left(1+\dfrac{\left(\Theta^{\overline{F\Re}_1}-1\right)^{\vartheta_1}}{(\Theta-1)^{\vartheta_1-1}}\right)\end{pmatrix}\right\}$$

$$\oplus_{F}\Box\left\{\begin{pmatrix}1-\log_\Theta\left(1+\dfrac{\left(\Theta^{1-\overline{TV}_2}-1\right)^{\vartheta_2}}{(\Theta-1)^{\vartheta_2-1}}\right),\\1-\log_\Theta\left(1+\dfrac{\left(\Theta^{1-\overline{T\Re}_2}-1\right)^{\vartheta_2}}{(\Theta-1)^{\vartheta_2-1}}\right)\end{pmatrix},\begin{pmatrix}\log_\Theta\left(1+\dfrac{\left(\Theta^{\overline{IV}_2}-1\right)^{\vartheta_2}}{(\Theta-1)^{\vartheta_2-1}}\right),\\\log_\Theta\left(1+\dfrac{\left(\Theta^{\overline{I\Re}_2}-1\right)^{\vartheta_2}}{(\Theta-1)^{\vartheta_2-1}}\right)\end{pmatrix},\begin{pmatrix}\log_\Theta\left(1+\dfrac{\left(\Theta^{\overline{FV}_2}-1\right)^{\vartheta_2}}{(\Theta-1)^{\vartheta_2-1}}\right),\\\log_\Theta\left(1+\dfrac{\left(\Theta^{\overline{F\Re}_2}-1\right)^{\vartheta_2}}{(\Theta-1)^{\vartheta_2-1}}\right)\end{pmatrix}\right\}$$

$$=\left\{\begin{pmatrix}1-\log_\Upsilon\left(1+\prod_{s=1}^{2}\left(\Upsilon^{1-\overline{TV}_j}-1\right)^{\vartheta_j}\right),\\1-\log_\Upsilon\left(1+\prod_{s=1}^{2}\left(\Upsilon^{1-\overline{T\Re}_j}-1\right)^{\vartheta_j}\right)\end{pmatrix},\begin{pmatrix}\log_\Upsilon\left(1+\prod_{s=1}^{2}\left(\Upsilon^{\overline{IV}_j}-1\right)^{\vartheta_j}\right),\\\log_\Upsilon\left(1+\prod_{s=1}^{2}\left(\Upsilon^{\overline{I\Re}_j}-1\right)^{\vartheta_j}\right)\end{pmatrix},\begin{pmatrix}\log_\Upsilon\left(1+\prod_{s=1}^{2}\left(\Upsilon^{\overline{FV}_j}-1\right)^{\vartheta_j}\right),\\\log_\Upsilon\left(1+\prod_{s=1}^{2}\left(\Upsilon^{\overline{F\Re}_j}-1\right)^{\vartheta_j}\right)\end{pmatrix}\right\}.$$

It is supposed that Eq.(9) holds for $n=n$, namely,

$$NZNFWA\left(\overline{NZ}_1,\overline{NZ}_2,\ldots,\overline{NZ}_n\right)=$$

$$\left\{\begin{pmatrix}1-\log_\Upsilon\left(1+\prod_{s=1}^{n}\left(\Upsilon^{1-\overline{TV}_j}-1\right)^{\vartheta_j}\right),\\1-\log_\Upsilon\left(1+\prod_{s=1}^{n}\left(\Upsilon^{1-\overline{T\Re}_j}-1\right)^{\vartheta_j}\right)\end{pmatrix},\begin{pmatrix}\log_\Upsilon\left(1+\prod_{s=1}^{n}\left(\Upsilon^{\overline{IV}_j}-1\right)^{\vartheta_j}\right),\\\log_\Upsilon\left(1+\prod_{s=1}^{n}\left(\Upsilon^{\overline{I\Re}_j}-1\right)^{\vartheta_j}\right)\end{pmatrix},\begin{pmatrix}\log_\Upsilon\left(1+\prod_{s=1}^{n}\left(\Upsilon^{\overline{FV}_j}-1\right)^{\vartheta_j}\right),\\\log_\Upsilon\left(1+\prod_{s=1}^{n}\left(\Upsilon^{\overline{F\Re}_j}-1\right)^{\vartheta_j}\right)\end{pmatrix}\right\}$$

For $n=n+1$, we have

$$NZNFWA\left(\overline{NZ}_1,\overline{NZ}_2,...,\overline{NZ}_{n+1}\right)=NZNFWA\left(\overline{NZ}_1,\overline{NZ}_2,...,\overline{NZ}_n\right)\oplus_F \vartheta_{n+1}\otimes_F \overline{NZ}_{n+1}$$

$$=\left\{\begin{array}{l}\left(\begin{array}{l}1-\log_\Upsilon\left(1+\prod_{s=1}^{n}\left(\Upsilon^{1-\overline{TV}_j}-1\right)^{\vartheta_j}\right),\\1-\log_\Upsilon\left(1+\prod_{s=1}^{n}\left(\Upsilon^{1-\overline{T\Re}_j}-1\right)^{\vartheta_j}\right)\end{array}\right),\left(\begin{array}{l}\log_\Upsilon\left(1+\prod_{s=1}^{n}\left(\Upsilon^{\overline{IV}_j}-1\right)^{\vartheta_j}\right),\\\log_\Upsilon\left(1+\prod_{s=1}^{n}\left(\Upsilon^{\overline{I\Re}_j}-1\right)^{\vartheta_j}\right)\end{array}\right),\left(\begin{array}{l}\log_\Upsilon\left(1+\prod_{s=1}^{n}\left(\Upsilon^{\overline{FV}_j}-1\right)^{\vartheta_j}\right),\\\log_\Upsilon\left(1+\prod_{s=1}^{n}\left(\Upsilon^{\overline{F\Re}_j}-1\right)^{\vartheta_j}\right)\end{array}\right)\end{array}\right\}$$

$$\oplus_F\left\{\begin{array}{l}\left(\begin{array}{l}1-\log_\Theta\left(1+\frac{\left(\Theta^{1-\overline{TV}_{n+1}}-1\right)^{\vartheta_{n+1}}}{\left(\Theta-1\right)^{\vartheta_{n+1}-1}}\right),\\1-\log_\Theta\left(1+\frac{\left(\Theta^{1-\overline{T\Re}_{n+1}}-1\right)^{\vartheta_{n+1}}}{\left(\Theta-1\right)^{\vartheta_{n+1}-1}}\right)\end{array}\right),\left(\begin{array}{l}\log_\Theta\left(1+\frac{\left(\Theta^{\overline{IV}_{n+1}}-1\right)^{\vartheta_{n+1}}}{\left(\Theta-1\right)^{\vartheta_{n+1}-1}}\right),\\\log_\Theta\left(1+\frac{\left(\Theta^{\overline{I\Re}_{n+1}}-1\right)^{\vartheta_{n+1}}}{\left(\Theta-1\right)^{\vartheta_{n+1}-1}}\right)\end{array}\right),\left(\begin{array}{l}\log_\Theta\left(1+\frac{\left(\Theta^{\overline{FV}_{n+1}}-1\right)^{\vartheta_{n+1}}}{\left(\Theta-1\right)^{\vartheta_{n+1}-1}}\right),\\\log_\Theta\left(1+\frac{\left(\Theta^{\overline{F\Re}_{n+1}}-1\right)^{\vartheta_{n+1}}}{\left(\Theta-1\right)^{\vartheta_{n+1}-1}}\right)\end{array}\right)\end{array}\right\}$$

$$=\left\{\begin{array}{l}\left(\begin{array}{l}1-\log_\Upsilon\left(1+\prod_{s=1}^{n+1}\left(\Upsilon^{1-\overline{TV}_j}-1\right)^{\vartheta_j}\right),\\1-\log_\Upsilon\left(1+\prod_{s=1}^{n+1}\left(\Upsilon^{1-\overline{T\Re}_j}-1\right)^{\vartheta_j}\right)\end{array}\right),\left(\begin{array}{l}\log_\Upsilon\left(1+\prod_{s=1}^{n+1}\left(\Upsilon^{\overline{IV}_j}-1\right)^{\vartheta_j}\right),\\\log_\Upsilon\left(1+\prod_{s=1}^{n+1}\left(\Upsilon^{\overline{I\Re}_j}-1\right)^{\vartheta_j}\right)\end{array}\right),\left(\begin{array}{l}\log_\Upsilon\left(1+\prod_{s=1}^{n+1}\left(\Upsilon^{\overline{FV}_j}-1\right)^{\vartheta_j}\right),\\\log_\Upsilon\left(1+\prod_{s=1}^{n+1}\left(\Upsilon^{\overline{F\Re}_j}-1\right)^{\vartheta_j}\right)\end{array}\right)\end{array}\right\}.$$

Hence, Eq. (9) keeps for $n=n+1$. Thus, Eq. (9) holds for all n, which finishes the proof of Theorem.

Proposition 1. Let $\overline{NZ}_j=\left\{\left(\overline{TV}_j,\overline{T\Re}_j\right),\left(\overline{IV}_j,\overline{I\Re}_j\right),\left(\overline{FV}_j,\overline{F\Re}_j\right)\right\}\left(j=1(1)n\right)$ be a set of SCFNs. Then NZNFWA operator has the following properties:

(1) Idempotency: If all SCFNs are equal, i.e., $\overline{NZ}_j=\overline{NZ}$, $\forall j$, then one has $NZNFWA\left(\overline{NZ}_1,\overline{NZ}_2,...,\overline{NZ}_n\right)=\overline{NZ}$.

(2) Boundedness: if $\overline{NZ}^-=\min\left(\overline{NZ}_1,\overline{NZ}_2,...,\overline{NZ}_n\right)$ and $\overline{NZ}^+=\max\left\{\overline{NZ}_1,\overline{NZ}_2,\cdots,\overline{NZ}_n\right\}$, then we have $\overline{NZ}^-\le NZNFWA\left(\overline{NZ}_1,\overline{NZ}_2,...,\overline{NZ}_n\right)\le\overline{NZ}^+$.

(3) Monotonicity: For the set of SCFNs, $\overline{NZ}_j$ and $\overline{NZ}_j\left(j=1,2,\cdots,n\right)$. If $\overline{NZ}_j\le\overline{NZ}_j$, then we have $NZNFWA\left(\overline{NZ}_1,\overline{NZ}_2,...,\overline{NZ}_n\right)\le NZNFWA\left(\overline{NZ}_1,\overline{NZ}_2,...,\overline{NZ}_n\right)$.

Definition 9. Let $\overline{NZ}_j=\left\{\left(\overline{TV}_j,\overline{T\Re}_j\right),\left(\overline{IV}_j,\overline{I\Re}_j\right),\left(\overline{FV}_j,\overline{F\Re}_j\right)\right\}\left(j=1(1)n\right)$ be a set of NZNs, ϑ_j is the weight of NZN $\overline{NZ}_j$ meeting $\sum_{j=1}^{n}\vartheta_j=1,\ \vartheta_j\in[0,1]$. Then Neutrosophic Z-number Frank ordered weighted averaging（NZNFOWA）operator is depicted as:

$$NZNFWA\left(\overline{NZ}_1,\overline{NZ}_2,...,\overline{NZ}_n\right)=\vartheta_1\otimes_F\overline{NZ}_{\iota(1)}\oplus_F\vartheta_2\otimes_F\overline{NZ}_{\iota(2)}\oplus_F\cdots\oplus_F\vartheta_n\otimes_F\overline{NZ}_{\iota(n)}.\qquad(10)$$

where $\left(\iota(1),\iota(2),\cdots,\iota(n)\right)$ stands for the permutation of $\left(j=1,2,\cdots,n\right)$ with $\overline{NZ}_{\iota(j-1)} \geq \overline{NZ}_{\iota(j)}$, $\forall j = 2,3,\cdots,n$.

Theorem 3. Let $\overline{NZ}_j = \left\{\left(\overline{TV}_j,\overline{T\Re}_j\right),\left(\overline{IV}_j,\overline{I\Re}_j\right),\left(\overline{FV}_j,\overline{F\Re}_j\right)\right\}\left(j=1(1)n\right)$ be a set of NZNs. Then the integration outcome of NZNs by NZNFOWA operator is a SCFN and represented as:

$$NZNFOWA\left(\overline{NZ}_1,\overline{NZ}_2,...,\overline{NZ}_n\right)=$$

$$\left\{\begin{pmatrix} 1-\log_{\Upsilon}\left(1+\prod_{s=1}^{n}\left(\Upsilon^{1-\overline{TV}_{\iota(j)}}-1\right)^{\vartheta_j}\right), \\ 1-\log_{\Upsilon}\left(1+\prod_{s=1}^{n}\left(\Upsilon^{1-\overline{T\Re}_{\iota(j)}}-1\right)^{\vartheta_j}\right) \end{pmatrix}, \begin{pmatrix} \log_{\Upsilon}\left(1+\prod_{s=1}^{n}\left(\Upsilon^{\overline{IV}_{\iota(j)}}-1\right)^{\vartheta_j}\right), \\ \log_{\Upsilon}\left(1+\prod_{s=1}^{n}\left(\Upsilon^{\overline{I\Re}_{\iota(j)}}-1\right)^{\vartheta_j}\right) \end{pmatrix}, \begin{pmatrix} \log_{\Upsilon}\left(1+\prod_{s=1}^{n}\left(\Upsilon^{\overline{FV}_{\iota(j)}}-1\right)^{\vartheta_j}\right), \\ \log_{\Upsilon}\left(1+\prod_{s=1}^{n}\left(\Upsilon^{\overline{F\Re}_{\iota(j)}}-1\right)^{\vartheta_j}\right) \end{pmatrix}\right\}. \tag{11}$$

Definition 10. Let $\overline{NZ}_j = \left\{\left(\overline{TV}_j,\overline{T\Re}_j\right),\left(\overline{IV}_j,\overline{I\Re}_j\right),\left(\overline{FV}_j,\overline{F\Re}_j\right)\right\}\left(j=1(1)n\right)$ be a set of NZNs, ϑ_j is the weight of NZN $\overline{NZ}_j$ meeting $\sum_{j=1}^{n}\vartheta_j = 1,\ \vartheta_j \in [0,1]$. Then Neutrosophic Z-number Frank weighted geometric（NZNFWG）operator is depicted as:

$$NZNFWG\left(\overline{NZ}_1,\overline{NZ}_2,...,\overline{NZ}_n\right)=\left(\overline{NZ}_1\right)^{\wedge_F \vartheta_1} \otimes_F \left(\overline{NZ}_2\right)^{\wedge_F \vartheta_2} \otimes_F \cdots \left(\overline{NZ}_n\right)^{\wedge_F \vartheta_n}. \tag{12}$$

Theorem 4. Let $\overline{NZ}_j = \left\{\left(\overline{TV}_j,\overline{T\Re}_j\right),\left(\overline{IV}_j,\overline{I\Re}_j\right),\left(\overline{FV}_j,\overline{F\Re}_j\right)\right\}\left(j=1(1)n\right)$ be a set of SCFNs. Then the integration outcome of NZNs by NZNFWG operator is a NZN and represented as:

$$NZNFWG\left(\overline{NZ}_1,\overline{NZ}_2,...,\overline{NZ}_n\right)=\left(\overline{NZ}_1\right)^{\wedge_F \vartheta_1} \otimes_F \left(\overline{NZ}_2\right)^{\wedge_F \vartheta_2} \otimes_F \cdots \left(\overline{NZ}_n\right)^{\wedge_F \vartheta_n}.$$

$$\left\{\begin{pmatrix} \log_{\Upsilon}\left(1+\prod_{s=1}^{n}\left(\Upsilon^{\overline{TV}_j}-1\right)^{\vartheta_j}\right), \\ \log_{\Upsilon}\left(1+\prod_{s=1}^{n}\left(\Upsilon^{\overline{T\Re}_j}-1\right)^{\vartheta_j}\right) \end{pmatrix}, \begin{pmatrix} 1-\log_{\Upsilon}\left(1+\prod_{s=1}^{n}\left(\Upsilon^{1-\overline{IV}_j}-1\right)^{\vartheta_j}\right), \\ 1-\log_{\Upsilon}\left(1+\prod_{s=1}^{n}\left(\Upsilon^{1-\overline{I\Re}_j}-1\right)^{\vartheta_j}\right) \end{pmatrix}, \begin{pmatrix} 1-\log_{\Upsilon}\left(1+\prod_{s=1}^{n}\left(\Upsilon^{1-\overline{FV}_j}-1\right)^{\vartheta_j}\right), \\ 1-\log_{\Upsilon}\left(1+\prod_{s=1}^{n}\left(\Upsilon^{1-\overline{F\Re}_j}-1\right)^{\vartheta_j}\right) \end{pmatrix}\right\}. \tag{13}$$

Proposition 2. Let $\overline{NZ}_j = \left\{\left(\overline{TV}_j,\overline{T\Re}_j\right),\left(\overline{IV}_j,\overline{I\Re}_j\right),\left(\overline{FV}_j,\overline{F\Re}_j\right)\right\}\left(j=1(1)n\right)$ be a set of SCFNs. Then NZNFWG operator has the following properties:

(1) Idempotency: If all SCFNs are equal, i.e., $\overline{NZ}_j = \overline{NZ}$, $\forall j$, then one has $NZNFWG\left(\overline{NZ}_1,\overline{NZ}_2,...,\overline{NZ}_n\right)=\overline{NZ}$.

(2) Boundedness: if $\overline{NZ}^{-} = \min\left(\overline{NZ}_1,\overline{NZ}_2,...,\overline{NZ}_n\right)$ and $\overline{NZ}^{+} = \max\left\{\overline{NZ}_1,\overline{NZ}_2,\cdots,\overline{NZ}_n\right\}$, then we have $\overline{NZ}^{-} \leq NZNFWG\left(\overline{NZ}_1,\overline{NZ}_2,...,\overline{NZ}_n\right) \leq \overline{NZ}^{+}$.

(3) Monotonicity: For the set of SCFNs, $\overline{NZ}_j$ and $\overline{NZ}_j\left(j=1,2,\cdots,n\right)$. If $\overline{NZ}_j \leq \overline{NZ}_j$, then we have $NZNFWG\left(\overline{NZ}_1,\overline{NZ}_2,...,\overline{NZ}_n\right) \leq NZNFWG\left(\overline{NZ}_1,\overline{NZ}_2,...,\overline{NZ}_n\right)$.

Definition 11. Let $\overline{NZ}_j = \left\{ \left(\overline{TV}_j, \overline{T\Re}_j \right), \left(\overline{IV}_j, \overline{I\Re}_j \right), \left(\overline{FV}_j, \overline{F\Re}_j \right) \right\} \left(j = 1(1)n \right)$ be a set of NZNs, ϑ_j is the weight of NZN $\overline{NZ}_j$ meeting $\sum_{j=1}^{n} \vartheta_j = 1,\ \vartheta_j \in [0, 1]$. Then Neutrosophic Z-number Frank ordered weighted geometric（NZNFOWG）operator is depicted as:

$$NZNFOWG\left(\overline{NZ}_1, \overline{NZ}_2, ..., \overline{NZ}_n\right) = \left(\overline{NZ}_{\iota(1)}\right)^{\wedge_F \vartheta_1} \otimes_F \left(\overline{NZ}_{\iota(2)}\right)^{\wedge_F \vartheta_2} \otimes_F \cdots \left(\overline{NZ}_{\iota(n)}\right)^{\wedge_F \vartheta_n}. \quad (14)$$

Where $\left(\iota(1), \iota(2), \cdots, \iota(n)\right)$ stands for the permutation of $\left(j = 1, 2, \cdots, n \right)$ with $\overline{NZ}_{\iota(j-1)} \geq \overline{NZ}_{\iota(j)}$, $\forall j = 2, 3, \cdots, n$.

Theorem 5. Let $\overline{NZ}_j = \left\{ \left(\overline{TV}_j, \overline{T\Re}_j \right), \left(\overline{IV}_j, \overline{I\Re}_j \right), \left(\overline{FV}_j, \overline{F\Re}_j \right) \right\} \left(j = 1(1)n \right)$ be a set of NZNs. Then the integration outcome of NZNs by NZNFOWG operator is a SCFN and represented as:

$$NZNFOWG\left(\overline{NZ}_1, \overline{NZ}_2, ..., \overline{NZ}_n\right) =$$

$$\left\{ \begin{pmatrix} \log_\Upsilon \left(1 + \prod_{s=1}^{n} \left(\Upsilon^{\overline{TV}_{\iota(j)}} - 1 \right)^{\vartheta_j} \right), \\ \log_\Upsilon \left(1 + \prod_{s=1}^{n} \left(\Upsilon^{\overline{T\Re}_{\iota(j)}} - 1 \right)^{\vartheta_j} \right) \end{pmatrix}, \begin{pmatrix} 1 - \log_\Upsilon \left(1 + \prod_{s=1}^{n} \left(\Upsilon^{1-\overline{IV}_{\iota(j)}} - 1 \right)^{\vartheta_j} \right), \\ 1 - \log_\Upsilon \left(1 + \prod_{s=1}^{n} \left(\Upsilon^{1-\overline{I\Re}_{\iota(j)}} - 1 \right)^{\vartheta_j} \right) \end{pmatrix}, \begin{pmatrix} 1 - \log_\Upsilon \left(1 + \prod_{s=1}^{n} \left(\Upsilon^{1-\overline{FV}_{\iota(j)}} - 1 \right)^{\vartheta_j} \right), \\ 1 - \log_\Upsilon \left(1 + \prod_{s=1}^{n} \left(\Upsilon^{1-\overline{F\Re}_{\iota(j)}} - 1 \right)^{\vartheta_j} \right) \end{pmatrix} \right\}. \quad (15)$$

4. Proposed methodology

In this section, we introduce a hybrid multi-criteria decision-making (MCDM) framework under neutrosophic Z-number environments, termed the NZN-LODECI-CoCoSo model, which integrates the Logarithmic Decomposition of Criteria Importance (LODECI) method with the Combined Compromise Solution (CoCoSo) technique. This approach is specifically designed to address decision problems where prior knowledge of criteria weights is unavailable. Within the proposed framework, experts articulate their uncertain evaluations of alternatives with respect to the predefined criteria using neutrosophic Z-numbers (NZNs). The objective weights of the criteria are derived through an enhanced LODECI method, which incorporates the score function of NZNs to improve discriminative capacity and robustness. Subsequently, the prioritization of alternatives is accomplished via an improved CoCoSo method tailored for NZN contexts, wherein the weighted sum and weighted product measures are computed using the neutrosophic Z-number fuzzy weighted averaging (NZNFWA) and neutrosophic Z-number fuzzy weighted geometric (NZNFWG) operators, respectively.

Formally, the MCDM problem under consideration can be delineated as follows: let $Q = \left\{ Q_i \middle| i = 1(1)m \right\}$ denote a discrete set of feasible alternatives, and let $P = \left\{ P_j \middle| j = 1(1)n \right\}$ represent a collection of evaluative attributes. The weight vector associated with the attributes is denoted by $\vartheta = \left\{ \vartheta_j \middle| j = 1(1)n \right\}$ satisfying $\omega\ \vartheta_j \in [0,1], \sum_{j=1}^{n} \vartheta_j = 1$. For each alternative $Q_i \left(i = 1, 2, \cdots, m \right)$ and attribute $P_j \left(j = 1, 2, \cdots, n \right)$, the performance is assessed via a neutrosophic Z-number, resulting in a decision matrix $\overline{K} = \left(\overline{NZ}_{ij} \right)_{m \times n}$, where $\overline{NZ}_{ij} = \left\{ \left(\overline{TV}_{ij}, \overline{T\Re}_{ij} \right), \left(\overline{IV}_{ij}, \overline{I\Re}_{ij} \right), \left(\overline{FV}_{ij}, \overline{F\Re}_{ij} \right) \right\}$ encapsulates the associated uncertain evaluation.

Based on the above notations, the procedural steps of the proposed NZN-LODECI-CoCoSo framework are elaborated as follows.

Step 1: Normalization of the decision matrix.

In MCDM, to eliminate the dimensional and magnitude discrepancies inherent in diverse attribute scales and to ensure commensurability, the original decision matrix must be normalized. This process converts cost-type criteria into benefit-type criteria. Consequently, the normalized decision matrix $\overline{\overline{K}} = \left(\overline{\overline{NZ}}_{ij}\right)_{m\times n}$ is derived using the following transformation equation:

$$\overline{\overline{NZ}}_{ij} = \begin{cases} \overline{NZ}_{ij} = \left\{\left(\overline{TV}_{ij}, \overline{T\Re}_{ij}\right), \left(\overline{IV}_{ij}, \overline{I\Re}_{ij}\right), \left(\overline{FV}_{ij}, \overline{F\Re}_{ij}\right)\right\}, j \in P_b \\ \left(\overline{NZ}_{ij}\right)^c = \left\{\left(\overline{FV}_{ij}, \overline{F\Re}_{ij}\right), \left(1-\overline{IV}_{ij}, 1-\overline{I\Re}_{ij}\right), \left(\overline{TV}_{ij}, \overline{T\Re}_{ij}\right)\right\}, j \in P_c \end{cases} \tag{16}$$

where P_b and P_c denote the benefit-type and cost-type criteria.

Step 2: Determination of criteria weights via the enhanced LODECI method

To objectively ascertain attribute weights without relying on subjective predispositions, we propose the NZN-LODECI method to attain the objective weight of criteria Pala[38]. The detailed procedural steps are as follows:

Step 2.1 Compute the score of normalized decision matrix.

The score matrix $F = \left[f_{ij}\right]_{m\times n}$ of normalized decision matrix can be computed by the following equation:

$$f_{ij} = SF\left(\overline{NZ}_j\right) = \frac{1}{6}\left(\begin{array}{l} 4 + \left(\overline{\overline{TV}}_{ij}\right)^2\left(\overline{\overline{T\Re}}_{ij}\right)^2 - \left(\overline{\overline{IV}}_{ij}\right)^2\left(\overline{\overline{I\Re}}_{ij}\right)^2 - \left(\overline{\overline{FV}}_{ij}\right)^2\left(\overline{\overline{F\Re}}_{ij}\right)^2 \\ + \overline{\overline{TV}}_{ij}\overline{\overline{T\Re}}_{ij} - \overline{\overline{IV}}_{j}\overline{\overline{I\Re}}_{j} - \overline{\overline{FV}}_{j}\overline{\overline{F\Re}}_{j} \end{array} \right) \tag{17}$$

Step 2.2 Compute the decomposition value SD_{ij} for each element in score matrix:

$$SD_{ij} = \max\left\{\left|f_{ij} - f_{rj}\right|\right\} r \neq i \tag{18}$$

Step 2.3 Calculate the logarithmic deviation value L_j for each criterion as follows:

$$L_j = \ln\left(1 + \frac{\sum_{i=1}^{m} SD_{ij}}{m}\right) \tag{19}$$

Step 2.4 Finally, the weight of each criterion is derived via:

$$\vartheta_j = \frac{L_j}{\sum_{j=1}^{n} L_j} \tag{20}$$

Step 3: Compute the weighted sum measure (WSM) and weighted product measure (WPM) for each alternative by NZNFWA operator and NZNFWG operator, respectively.

$$\rho_i = NZNFWA\left(\overline{\overline{NZ}}_{i1}, \overline{\overline{NZ}}_{i2}, \ldots, \overline{\overline{NZ}}_{in}\right) =$$

$$\left\{ \begin{pmatrix} 1 - \log_\Upsilon\left(1 + \prod_{j=1}^{n}\left(\Upsilon^{1-\overline{\overline{TV}}_{ij}} - 1\right)^{\vartheta_j}\right), \\ 1 - \log_\Upsilon\left(1 + \prod_{j=1}^{n}\left(\Upsilon^{1-\overline{\overline{T\Re}}_{ij}} - 1\right)^{\vartheta_j}\right) \end{pmatrix}, \begin{pmatrix} \log_\Upsilon\left(1 + \prod_{j=1}^{n}\left(\Upsilon^{\overline{\overline{IV}}_{ij}} - 1\right)^{\vartheta_j}\right), \\ \log_\Upsilon\left(1 + \prod_{j=1}^{n}\left(\Upsilon^{\overline{\overline{I\Re}}_{ij}} - 1\right)^{\vartheta_j}\right) \end{pmatrix}, \begin{pmatrix} \log_\Upsilon\left(1 + \prod_{j=1}^{n}\left(\Upsilon^{\overline{\overline{FV}}_{ij}} - 1\right)^{\vartheta_j}\right), \\ \log_\Upsilon\left(1 + \prod_{j=1}^{n}\left(\Upsilon^{\overline{\overline{F\Re}}_{ij}} - 1\right)^{\vartheta_j}\right) \end{pmatrix} \right\} \tag{21}$$

$$\sigma_i = NZNFWG\left(\overline{\overline{NZ}}_{i1}, \overline{\overline{NZ}}_{i2}, \ldots, \overline{\overline{NZ}}_{in}\right) = .$$

$$\left\{\begin{pmatrix} \log_{\Upsilon}\left(1+\prod_{j=1}^{n}\left(\Upsilon^{\overline{\overline{TV}}_{ij}}-1\right)^{\vartheta_j}\right), \\ \log_{\Upsilon}\left(1+\prod_{j=1}^{n}\left(\Upsilon^{\overline{\overline{T\Re}}_{ij}}-1\right)^{\vartheta_j}\right) \end{pmatrix}, \begin{pmatrix} 1-\log_{\Upsilon}\left(1+\prod_{j=1}^{n}\left(\Upsilon^{1-\overline{\overline{IV}}_{ij}}-1\right)^{\vartheta_j}\right), \\ 1-\log_{\Upsilon}\left(1+\prod_{j=1}^{n}\left(\Upsilon^{1-\overline{\overline{I\Re}}_{ij}}-1\right)^{\vartheta_j}\right) \end{pmatrix}, \begin{pmatrix} 1-\log_{\Upsilon}\left(1+\prod_{j=1}^{n}\left(\Upsilon^{1-\overline{\overline{FV}}_{ij}}-1\right)^{\vartheta_j}\right), \\ 1-\log_{\Upsilon}\left(1+\prod_{j=1}^{n}\left(\Upsilon^{1-\overline{\overline{F\Re}}_{ij}}-1\right)^{\vartheta_j}\right) \end{pmatrix}\right\} \quad (22)$$

Step 4: Transform the aggregated NZN values ρ_i and σ_i into crisp scores via the NZN score function $SF(\rho_i)$ and $SF(\sigma_i)$, displayed as:

$$\wp_i = SF(\rho_i), \quad (23)$$

$$\Im_i = SF(\sigma_i), \quad (24)$$

where $SF\left(\overline{NZ}\right) = \frac{1}{6}\left(4 + \left(\overline{TV}\right)^2\left(\overline{T\Re}\right)^2 - \left(\overline{IV}\right)^2\left(\overline{I\Re}\right)^2 - \left(\overline{FV}\right)^2\left(\overline{F\Re}\right)^2 + \overline{TV}\,\overline{T\Re} - \overline{IV}\,\overline{I\Re} - \overline{FV}\,\overline{F\Re}\right)$ for a NZN $\overline{NZ} = \left\{\left(\overline{TV}, \overline{T\Re}\right), \left(\overline{IV}, \overline{I\Re}\right), \left(\overline{FV}, \overline{F\Re}\right)\right\}$.

Step 5: The relative importance degree of alternatives shall be ascertained with the aid of three appraisal score strategies as below:

$$D_i^1 = \frac{\wp_i + \Im_i}{\sum_{i=1}^{m}\left(\wp_i + \Im_i\right)}, \quad (25)$$

$$D_i^2 = \frac{\wp_i}{\left(1 + \min_i \wp_i\right)} + \frac{\Im_i}{\left(1 + \min_i \Im_i\right)}, \quad (26)$$

$$D_i^3 = \frac{o\wp_i + (1-o)\Im_i}{o\max_i \wp_i + (1-o)\max_i \Im_i}, o \in [0,1]. \quad (27)$$

Step 6: Compute the final assessment index D_i by synthesizing the three strategies:

$$D_i = \left(D_i^1 D_i^2 D_i^3\right)^{\frac{1}{3}} + \frac{1}{3}\left(D_i^1 + D_i^2 + D_i^3\right) \quad (28)$$

Step 7: Rank the alternatives in descending order of final assessment index D_i . The optimal alternative has highest assessment index D_i .

5. Case study

As a critical direction for achieving sustainable and high-quality development in the logistics sector, green logistics not only contributes to the national sustainable development strategy but also pursues the dual goals of economic growth and environmental protection. As a major economic center for manufacturing and industrial development in China, a region still faces certain limitations in its logistics development. To further enhance the quality of logistics development in the region, an expert team was invited to conduct a comprehensive evaluation of the green logistics development levels in its four counties. Based on preliminary research and analysis, the expert team established five evaluation indicators: Infrastructure Investment (P_1), Logistics Demand(P_2), Operations Management(P_3), Environmental and Resource Investment Costs(P_4) and Innovation Development

Level(P_5). Considering the cognitive uncertainty in expert evaluations, this study employs NZN to represent the assessment values of the four counties under each of the six indicators, thereby constructing the evaluation matrix, as shown in Table 1.

5.1 Method implementation

In the following, we utilize the proposed NZN- LODECI-CoCoSo method to handle this decision problem.

Table 1
Neutrosophic Z-number evaluation matrix

	P_1	P_2	P_3	P_4	P_5
Q_1	{(0.6,0.4), (0.9,0.5), (0.6,0.4)}	{(0.3,0.3), (0.5,0.5), (0.8,0.4)}	{(0.5,0.4), (0.5,0.3), (0.4,0.2)}	{(0.6,0.45), (0.4,0.3), (0.5,0.2)}	{(0.5,0.4), (0.5,0.3), (0.7,0.4)}
Q_2	{(0.7,0.4), (0.5,0.4), (0.4,0.3)}	{(0.6,0.4), (0.7,0.4), (0.3,0.2)}	{(0.8,0.2), (0.4,0.3), (0.4,0.2)}	{(0.5,0.4), (0.8,0.5), (0.6,0.4)}	{(0.6,0.5), (0.7,0.4), (0.3,0.2)}
Q_3	{(0.6,0.4), (0.7,0.4), (0.3,0.2)}	{(0.5,0.45), (0.5,0.3), (0.6,0.3)}	{(0.7,0.5), (0.6,0.3), (0.3,0.2)}	{(0.4,0.45), (0.7,0.4), (0.7,0.3)}	{(0.5,0.4), (0.6,0.3), (0.6,0.3)}
Q_4	{(0.6,0.3), (0.5,0.5), (0.5,0.2)}	{(0.65,0.4), (0.5,0.3), (0.6,0.4)}	{(0.7,0.3), (0.8,0.3), (0.3,0.2)}	{(0.8,0.45), (0.7,0.65), (0.5,0.5)}	{(0.8,0.3), (0.6,0.3), (0.7,0.3)}
Q_5	{(0.8,0.45), (0.7,0.35), (0.5,0.2)}	{(0.7,0.4), (0.8,0.3), (0.6,0.3)}	{(0.6,0.4), (0.5,0.4), (0.6,0.2)}	{(0.4,0.3), (0.3,0.4), (0.7,0.3)}	{(0.4,0.45), (0.75,0.45), (0.8,0.4)}

Step 1: Normalization of the decision matrix. Because the Environmental and Resource Investment Costs (P_4) index is the cost-type criteria, the normalized Neutrosophic Z-number evaluation matrix is attained by Eq. (16), the results are listed in Table2.

Table 2
Normalized Neutrosophic Z-number evaluation matrix

	P_1	P_2	P_3	P_4	P_5
Q_1	{(0.6,0.4), (0.9,0.5), (0.6,0.4)}	{(0.3,0.3), (0.5,0.5), (0.8,0.4)}	{(0.5,0.4), (0.5,0.3), (0.4,0.2)}	{(0.5,0.2), (0.6,0.7), (0.6,0.45)}	{(0.5,0.4), (0.5,0.3), (0.7,0.4)}
Q_2	{(0.7,0.4), (0.5,0.4), (0.4,0.3)}	{(0.6,0.4), (0.7,0.4), (0.3,0.2)}	{(0.8,0.2), (0.4,0.3), (0.4,0.2)}	{(0.6,0.4), (0.2,0.5), (0.5,0.4)}	{(0.6,0.5), (0.7,0.4), (0.3,0.2)}
Q_3	{(0.6,0.4), (0.7,0.4), (0.3,0.2)}	{(0.5,0.45), (0.5,0.3), (0.6,0.3)}	{(0.7,0.5), (0.6,0.3), (0.3,0.2)}	{(0.7,0.3), (0.3,0.6), (0.4,0.45),}	{(0.5,0.4), (0.6,0.3), (0.6,0.3)}
Q_4	{(0.6,0.3), (0.5,0.5), (0.5,0.2)}	{(0.65,0.4), (0.5,0.3), (0.6,0.4)}	{(0.7,0.3), (0.8,0.3), (0.3,0.2)}	{(0.5,0.5), (0.3,0.35), (0.8,0.45)}	{(0.8,0.3), (0.6,0.3), (0.7,0.3)}

Q_5	{(0.8,0.45), (0.7,0.35), (0.5,0.2)}	{(0.7,0.4), (0.8,0.3), (0.6,0.3)}	{(0.6,0.4), (0.5,0.4), (0.6,0.2)}	{(0.7,0.3), (0.7,0.3), (0.4,0.3),}	{(0.4,0.45), (0.75,0.45), (0.8,0.4)}

Step 2: Determination of criteria weights via the enhanced LODECI method

Step 2.1 The score of normalized Neutrosophic Z-number evaluation matrix is determined by Eq. (17) and shown as Table3.

Table 3
The score of normalized Neutrosophic Z-number evaluation matrix

	P_1	P_2	P_3	P_4	P_5
Q_1	0.5579	0.5605	0.6635	0.5285	0.6182
Q_2	0.6640	0.6459	0.6608	0.6579	0.6613
Q_3	0.6459	0.6485	0.6994	0.6382	0.6359
Q_4	0.6317	0.6429	0.6488	0.6178	0.6385
Q_5	0.6791	0.6414	0.6539	0.6443	0.5452

Step 2.2. The decomposition value SD_{ij} for each element in score matrix is computed by Eq.(18) and listed in Table 4 .

Table 4
The decomposition value SD_{ij}

	P_1	P_2	P_3	P_4	P_5
Q_1	0.1212	0.0879	0.0359	0.1295	0.0730
Q_2	0.5911	0.5605	0.6488	0.5345	0.5479
Q_3	0.0880	0.0879	0.0506	0.1098	0.0906
Q_4	0.0737	0.0824	0.0506	0.0894	0.0933
Q_5	0.1212	0.0809	0.0456	0.1158	0.1161

Step 2.3-2.5. The logarithmic deviation value L_j for each criterion is calculated by Eq.(19) and he weight of each criterion is derived by Eq.(20), and listed as $\vartheta_1 = 0.2139$, $\vartheta_2 = 0.1950$, $\vartheta_3 = 0.1813$, $\vartheta_4 = 0.2107$, $\vartheta_5 = 0.1992$.

Step 3: The WSM and WPM for each alternative by Eq. (21) and Eq. (22), respectively. The results are displayed as

$$\rho_1 = \{(0.4895, 0.3421), (0.5926, 0.4455), (0.6109, 0.3631)\},$$

$$\rho_2 = \{(0.6671, 0.3891), (0.4588, 0.3986), (0.3752, 0.2534)\},$$

$$\rho_3 = \{(0.6085, 0.4097), (0.5208, 0.3711), (0.4217, 0.2797)\},$$

$$\rho_4 = \{(0.6605, 0.3661), (0.5112, 0.3466), (0.5628, 0.2961)\},$$

$$\rho_5 = \{(0.6646, 0.4016), (0.6870, 0.3547), (0.5638, 0.2783)\},$$

$$\sigma_1 = \{(0.4724, 0.3281), (0.6557, 0.4858), (0.6422, 0.3783)\},$$
$$\sigma_2 = \{(0.6542, 0.3706), (0.5279, 0.4056), (0.3855, 0.2672)\},$$
$$\sigma_3 = \{(0.5944, 0.4019), (0.5552, 0.3954), (0.4535, 0.2972)\},$$
$$\sigma_4 = \{(0.6403, 0.3543), (0.5604, 0.3576), (0.6167, 0.3177)\},$$
$$\sigma_5 = \{(0.6300, 0.3958), (0.7054, 0.3609), (0.5986, 0.2953)\}.$$

Step 6: The score of the aggregated NZN values ρ_i and σ_i is computed by Eq. (23) and Eq. (24), respectively. The results are displayed as: $\wp_1 = 0.5985$, $\wp_2 = 0.6678$, $\wp_3 = 0.6582$, $\wp_4 = 0.6495$, $\wp_5 = 0.6423$, $\Im_1 = 0.5762$, $\Im_2 = 0.6546$, $\Im_3 = 0.6459$, $\Im_4 = 0.6339$, $\Im_5 = 0.6307$.

Step 7-9: The relative importance degree of alternatives can be ascertained via t Eqs. (25)-(28), the results are displayed in Table 5. The ranking of alternatives is attained in descending order of final assessment index D_i, namely, $Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$.

Table 5
The results of relative importance degrees for each alternative

	D_i^1	Ranking	D_i^2	Ranking	D_i^3	Ranking	D_i	Ranking
Q_1	0.1848	5	0.7400	5	0.8883	5	1.0995	5
Q_2	0.2080	1	0.8331	1	1.0000	1	1.2379	1
Q_3	0.2051	2	0.8215	2	0.9862	2	1.2207	2
Q_4	0.2019	3	0.8085	3	0.9706	3	1.2014	3
Q_5	0.2002	4	0.8019	4	0.9626	4	1.1916	4

5.2 Sensitivity analysis

To examine the stability and robustness of the proposed NZN-LODECI-CoCoSo decision method, this section investigates the impact of two critical parameters: the parameter Υ in the Frank operator, which depicts the risk preference of decision expert, and the parameter o in the CoCoSo method, which adjusts the balance of WSM and WPM in the third fusion strategy in CoCoSo method .

Case 1: Effect of the parameter Υ in the Frank operator. The influence of the Frank operator parameter Υ was examined by incrementally varying its value from 2 to 20, the relative importance degree scores and rankings for each alternative are summarized in Table 6 and Figure 1.

As shown in the Table 6 and Figure 1, the relative importance degree scores for all alternatives remain highly stable based on diverse value of parameter Υ. We can find that Q_2 is the best choice during the discussed results, which indicates that the proposed NZN-LODECI-CoCoSo decision is insensitive to the parameter Υ, Hence, the propounded method ha stronger robustness and stability.

Table 6
Ranking results based on different values of parameter Υ

Υ	Q_1	Q_2	Q_3	Q_4	Q_5	Rank
2	1.0995	1.2379	1.2207	1.2014	1.1916	$Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$
3	1.0998	1.2378	1.2207	1.2014	1.1916	$Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$
4	1.1000	1.2377	1.2207	1.2014	1.1916	$Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$
5	1.1001	1.2377	1.2207	1.2014	1.1916	$Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$
7	1.1003	1.2377	1.2206	1.2014	1.1916	$Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$
10	1.1005	1.2376	1.2206	1.2013	1.1916	$Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$
15	1.1007	1.2375	1.2206	1.2013	1.1915	$Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$
20	1.1008	1.2375	1.2205	1.2013	1.1915	$Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$

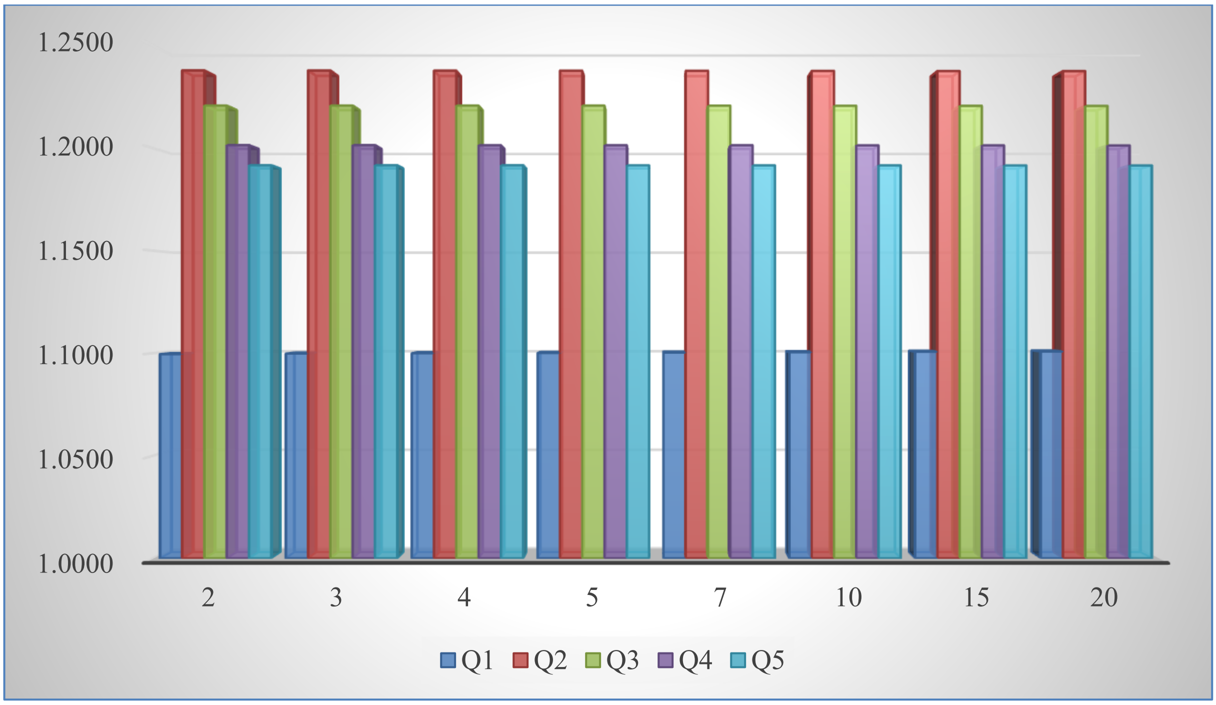

Fig.1. The change of relative importance degree for all alternatives over diver values of parameter Υ

Case 2: Effect of the parameter o in the CoCoSo method. To evaluate the sensitivity of parameter o in the CoCoSo method, we systematically adjust parameter o from 0 to 1 with a step size of 0.1. The corresponding results are presented in the accompanying Table 7 and Figure 2. Based on the attained results, the ranking of the alternatives remains fixed, following the order $Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$. While the relative importance degree scores display minor monotonic shifts, for instance, the score of Q_2 exhibits a slight upward trend as o increases, but these fluctuations fail to affect the final ordering. This consistency shows that the decision outcomes produced by the CoCoSo method are stable with respect to the choice of o, enhancing the reliability of the proposed NZN-LODECI-CoCoSo decision framework.

Table 7
Ranking results based on different values of parameter o

o	Q_1	Q_2	Q_3	Q_4	Q_5	Rank
0.0	1.0953	1.2379	1.2210	1.2003	1.1920	$Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$
0.1	1.0962	1.2379	1.2209	1.2005	1.1920	$Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$
0.2	1.0970	1.2379	1.2209	1.2008	1.1919	$Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$
0.3	1.0979	1.2379	1.2208	1.2010	1.1918	$Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$
0.4	1.0987	1.2379	1.2208	1.2012	1.1917	$Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$
0.5	1.0995	1.2379	1.2207	1.2014	1.1916	$Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$
0.6	1.1004	1.2379	1.2207	1.2017	1.1915	$Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$
0.7	1.1012	1.2379	1.2206	1.2019	1.1914	$Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$
0.8	1.1020	1.2379	1.2206	1.2021	1.1914	$Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$
0.9	1.1029	1.2379	1.2205	1.2023	1.1913	$Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$
1.0	1.1037	1.2379	1.2205	1.2026	1.1912	$Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$

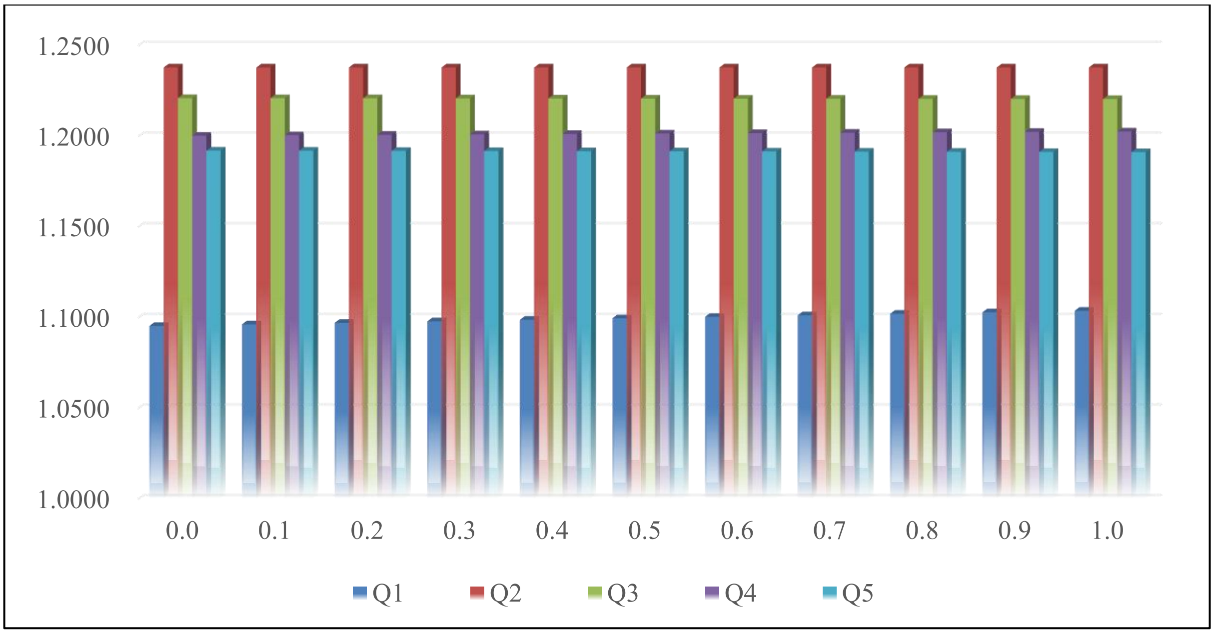

Fig.2. The change of relative importance degree for all alternatives over diver values of parameter Υ

5.3 Comparison analysis

To validate the effectiveness of the proposed NZN-LODECI-CoCoSo method in determining the optimal alternative, a comparative analysis was conducted between the proposed method and previous approaches, including the NZNWA operator-based method, NZNWG operator-based method, NZNAAWA operator-based method and NZNSSPWA operator. The comparison outcomes of the compared methods are displayed in Table 8 and Figure 3.

Table 8
Decision outcome based on different NZN-MADM methods

	Overall evaluation measure					Ranking
	Q_1	Q_2	Q_3	Q_4	Q_5	
NZNWA operator-based method[13]	0.5627	0.6622	0.6476	0.6346	0.6234	$Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$
NZNWG operator-based method[13]	0.5283	0.6402	0.6265	0.6082	0.6045	$Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$
NZNAAWA operator-based method[15]	0.6186	0.7477	0.7142	0.7009	0.7049	$Q_2 \succ Q_3 \succ Q_5 \succ Q_4 \succ Q_1$
NZNAAWG operator-based method[15]	0.4262	0.6171	0.6080	0.5810	0.5430	$Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$
Proposed method	1.0995	1.2379	1.2207	1.2014	1.1916	$Q_2 \succ Q_3 \succ Q_4 \succ Q_5 \succ Q_1$

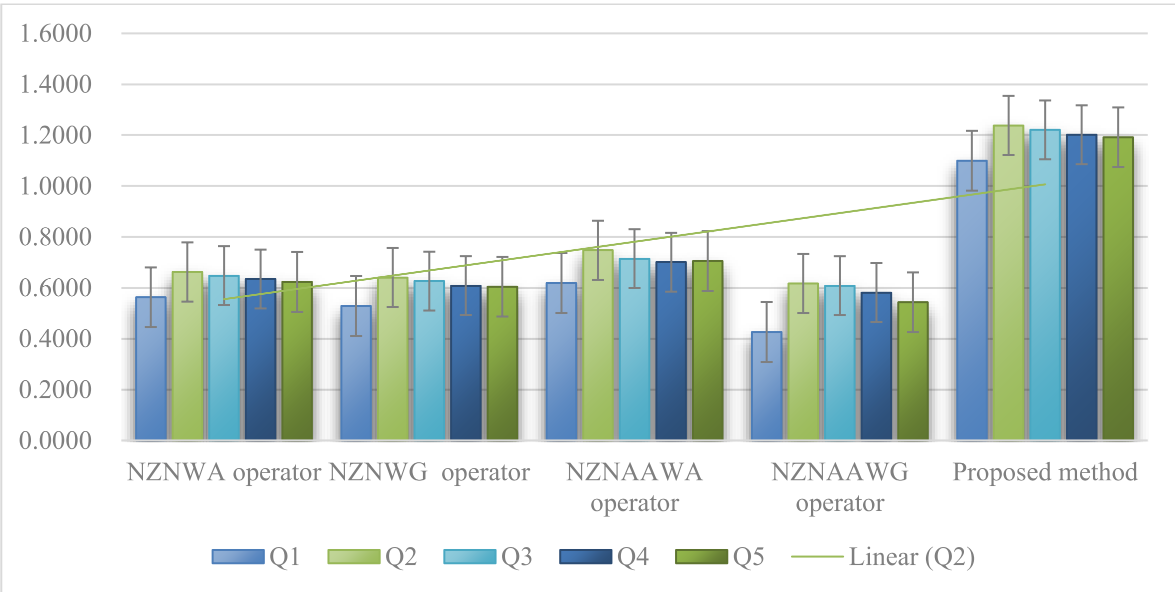

Fig.3. Decision outcome based on different NZN-MADM methods

From the comparison results, it can be observed that the ranking orders derived from most of the compared methods are consistent with that of the proposed NZN-LODECI-CoCoSo method, with the exception of the NZNAAWA operator-based approach. Moreover, the optimal alternative is consistently identified as Q_2 across all valid methods, which further confirms the consistency and effectiveness of the proposed framework.

A detailed analysis of the compared NZN-based multi-attribute decision-making (MADM) approaches is provided as follows. The four benchmark methods are all constructed upon aggregation operators; however, they fail to simultaneously account for both the weighted product model (WPM) and the weighted sum model (WSM) within a unified framework. Furthermore, the score functions employed in the NZNWA operator-based method[13], NZNWG operator-based method[13], NZNAAWA operator-based method[15], and NZNAAWG operator-based method[15] exhibit inherent limitations, which may lead to unreasonable ranking outcomes under certain conditions. In contrast, the proposed NZN-LODECI-CoCoSo method incorporates a novel score function that enhances the reliability of the final ranking results. Additionally, by integrating Frank operators into the CoCoSo framework, the proposed method not only increases the flexibility of the aggregation process but also strengthens the robustness of the decision outcomes through the synergistic use of three distinct fusion strategies. Moreover, the NZN-LODECI approach enables objective determination of criteria weights, thereby rendering the overall decision-making process more applicable to practical scenarios.

Collectively, the proposed NZN-LODECI-CoCoSo method integrates the LODECI, CoCoSo method and Frank aggregation operator. The combination of Frank operators with the CoCoSo framework facilitates a more powerful information aggregation while maintaining stability against parameter variations. These features enable the proposed method to yield more distinct and reliable rankings over the compared approaches.

6. Conclusions

This study has developed a novel MADM framework under uncertain environments by integrating NZN with the CoCoSo method, Frank aggregation operators, and the ODECI model. The proposed approach addresses key limitations in existing NZN-based decision methodologies by enhancing the flexibility of information aggregation, improving the objectivity of attribute weight determination, and ensuring robust compromise ranking of alternatives. The main contributions of this paper are summarized as follows. First, we defined operational rules for NZNs based on Frank triangular norms, which provide a flexibility for information fusion. Building upon these operations, we proposed the NZNFWA and NZNFWG operators, along with their ordered weighted operators. Second, we extended the LODECI model to the NZN context by incorporating the score function to attain objective weight of attribute. Third, we constructed the NZN-LODECI-CoCoSo decision framework, which integrates the proposed aggregation operators into the enhanced CoCoSo method to derive stable and reliable prioritization of alternatives. The applicability and effectiveness of the proposed NZN-LODECI-CoCoSo method were validated through a case study evaluating green logistics development levels. Parameter analysis demonstrated that the proposed framework exhibits stability under varying parameter settings. Comparative discussions with existing NZN-based approaches further highlighted the superiority of the proposed NZN-LODECI-CoCoSo method.

Future research directions may explore the integration of NZNs with other advanced decision models and novel aggregation operators such as Frank Muirhead mean, generalized Dombi Bonferroni mean and so forth. We also devote to research the NZN-based group decision methodology based on social network, consensus reaching process and truth relationship.

Author Contributions

For research articles with several authors, a short paragraph specifying their individual contributions must be provided. The following statements should be used "Conceptualization, D. Li. and Y. Rong; methodology, D. Li. and Y. Rong; software, D. Li. formal analysis, Y. Rong; investigation, D. Li. and Y. Rong; writing—original draft preparation, D. Li. and Y. Rong; writing—review and editing, D. Li. and Y. Rong; funding acquisition, D. Li. All authors have read and agreed to the published version of the manuscript." Authorship must be limited to those who have contributed substantially to the work reported.

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Data availability

The authors confirm that the data supporting the findings of this study are available within the article.

Declarations

I hereby and on behalf of the co-authors, declare all the authors agreed to submit the article exclusively to this journal and also declare that there is no Conflict of interest regarding the publication of this article.

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