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Dynamic intuitionistic fuzzy soft set-based WASPAS model for multi-criteria decision making in renewable energy selection

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ABSTRACT

The nature of the multi-criteria decision-making problem is full of uncertainty in the expert decisions of the task as well as the changes in the technological performance over time, which represent the other characteristics of renewable energy planning. Nevertheless, several of the available fuzzy multi-criteria decision-making solutions are not dynamic in change and they fail to adapt to change over time. In order to overcome this weakness, this paper suggests a dynamic intuitionistic fuzzy soft set based WASPAS model to assess the renewable energy alternatives under uncertainty and time dynamics. The proposed method would incorporate the intuitionistic fuzzy soft set modeling and the WASPAS aggregation protocol in an attempt to model expert hesitation as well as synthesize multi-year appraisals by a recency-based temporal weighting. To show how the framework could be applied, a case study of the renewable energy planning in Pakistan is performed. There are four options that include solar, wind, hydropower, and bioenergy, which are analyzed in eight economic, environmental, and technical parameters, based on the expert ratings of 2021-2023. The findings show that the hydropower gives the best overall performance, then the wind, the solar, and the bioenergy. Sensitivity and comparative analyses also prove the viability and solidity of the obtained rankings. The advanced dynamic intuitionistic fuzzy soft set based WASPAS framework is a powerful decision-support system in the renewable energy planning process in the uncertainties and dynamic conditions and can be applied in other multi-criteria decision-making processes whereby the criteria and expert preference changes with time.

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1. Introduction

Renewable energy use has now become a critical approach to solving the growing energy demand, environmental degradation [19], and energy security issues in the global arena. The need to have sustainable energy planning has been heightened by rapid growth in population, pressures on climatic changes, and exhaustion of fossil fuel reserves. These are very apparent in the developing world like Pakistan, where the problem of electricity deficiency, excessive reliance on fossil fuels imported, and poor use of renewable energy sources [20] is still problematic. Even though renewable energy like solar, wind, hydro and bio-energy have a lot of potential, the choice of the most suitable alternative is a complicated issue of choice.

The choice of renewable energy can be developed to be a multi-criteria decision-making (MCDM) problem where economic, environmental, technical, and operational criteria, which are usually conflicting, have to be considered simultaneously. Moreover, the criteria [5] of the decision-making are generally uncertainties, subjective, and can be hesitating. In the real world energy planning, these standards and preferences of experts are not fixed and they change as time goes by as a result of technological developments, policy shifts, market flexibilities and environmental factors. The effective decision-making models should therefore have the ability to provide intuitionistic uncertainty as well as temporal variation.

MCDM procedures have also undergone the development to deal with complex decision-making situations in multi-criteria situations of conflicting criteria. Classical methods include the Analytic Hierarchy Process (AHP) [25] and the Technique for order preference by similarity to ideal solution (TOPSIS) [6] which were developed to form the basics of decisions-making through pair-wise comparisons and distance to ideal solution. Other popular methods like ELECTRE [15] described the outranking relation concept whereas PROMETHEE [3] identified the preference flow analysis of ranking the alternatives. The most favored of these methods is the Weighted Aggregated Sum Product Assessment (WASPAS) method which has proved to be an effective combined method of decision making as it incorporates weighted sum model and weighted product model into the combined model thereby enhancing the stability and accuracy of the ranking results obtained.

Table 1 is the overview of the significant progress and theoretical background in fuzzy set-based and soft set-based decision making models. It gives importance to the methodologies followed, primary contribution of each research and the respective gaps in research that led to the construction of proposed framework the Dynamic Intuitionistic Fuzzy Soft Set based WASPAS (DIFSS-WASPAS). The studies in reference all show how the uncertainty, parameterization and dynamic aspects have been developed in various extensions of fuzziness and soft computing paradigms.

Table 1: Summary of Related Works on Fuzzy and Soft-Set-Based Decision-Making Frameworks

Author(s)	**Methodology**	**Main Contribution**	**Limitations / Research Gap**
Zadeh [28]	Fuzzy Set Theory	Introduced fuzzy sets for modeling vagueness and partial truth.	Unable to capture hesitation or parameterization.
Atanassov [2]	Intuitionistic Fuzzy Set	Added membership, non-membership, and hesitation degrees.	No temporal or parametric considerations.
Molodtsov [12]	Soft Set Theory	Introduced parameterization in decision problems.	Does not include fuzziness.
Maji et al. [10]	Fuzzy Soft Set	Combined soft sets and fuzzy sets for uncertain decision representation.	Lacks hesitation and temporal adaptability.
Maji et al. [9]	Intuitionistic Fuzzy Soft Set	Integrated intuitionistic fuzziness with soft parameters.	Static in nature; no dynamic modeling.
Saqlain et al. [21]	Cubic Intuitionistic Fuzzy Soft Set	Developed distance measures for decision-making.	Does not address time-dependent systems.
Saeed et al. [18]	Cubic Pythagorean Fuzzy Soft Set	Extended interval-valued fuzzy sets to cubic soft structures.	Dynamic components absent.
Saeed et al. [17]	Cubic Intuitionistic Fuzzy Hypersoft Set	Defined Hausdorff and Hamming distances for industrial evaluation.	Application limited; lacks temporal variation.
Rahman et al. [13]	Hypersoft Set Framework	Developed medical decision support for tumor vulnerability assessment.	Focused on specific domain; no MCDM integration.
Arshad et al. [1]	Interval-Valued Multi-Fuzzy Hypersoft Set	Designed optimal mask selection model during COVID-19.	Static model; lacks temporal evolution.
Turskis et al. [24]	Fuzzy WASPAS	Incorporated fuzzy numbers into WASPAS for uncertainty handling.	Assumes static evaluation criteria.
Rani et al. [14]	Intuitionistic Fuzzy WASPAS	Embedded intuitionistic fuzzy environment into WASPAS.	Time-dependent variations not addressed.
Li et al. [8]	Soft Set–Based WASPAS	Introduced parameterization into WASPAS for diversified decisions.	No dynamic adaptability.
Masoomi et al. [11]	Fuzzy AHP and TOPSIS	Applied fuzzy MCDM methods in renewable energy planning.	Static models ignoring temporal progress.
Shi et al. [22]	Review of Fuzzy MCDM	Surveyed fuzzy MCDM approaches in renewable energy planning.	Predominantly static formulations.
Xu et al. [26]	Dynamic MCDM Models	Proposed dynamic decision-making using numerical data.	Does not integrate fuzzy uncertainty.
Yolcu et al. [27]	Intuitionistic Fuzzy Hypersoft Set	Combined intuitionistic fuzziness with hypersoft structure.	No linkage with hybrid aggregation like WASPAS.
Tao et al. [23]	Intuitionistic Fuzzy Group Decision Model	Modeled dynamic group decision using fuzzy aggregation operators.	Temporal and parameter variations not fully captured.
Kannan et al. [7]	Pythagorean Fuzzy Soft WASPAS	Enhanced WASPAS with Pythagorean fuzzy soft structures.	Unified dynamic-uncertain modeling absent.
Saeed [16]	Dynamic Soft Set (DSS)	Proposed DSS with time-varying parameters for dynamic decisions.	Lacks integration with intuitionistic fuzziness and aggregation mechanisms.
Proposed Study	**DIFSS-WASPAS**	**Integrates intuitionistic fuzziness, soft parameterization, and temporal modeling within WASPAS for renewable energy evaluation in Pakistan.**	**Bridges existing gaps through unification of dynamic, uncertain, and multi-parameter decision frameworks.**

Motivation

The issue of renewable energy planning has emerged as an issue of major concern to the policy makers and energy planners with the growing global focus on sustainable developments and environmental protection. Nevertheless, uncertainty, subjective analysis of experts and shifting criteria usually affect the assessment of renewable energy options. These issues complicate and make the decision-making process complicate and hard to resolve with the help of traditional models.
Existing decision-making frameworks are largely not robust enough to be able to deal with dynamic behavior as well as intuitionistic uncertainty at the same time. This weakness stimulates the creation of a superior decision-supporting methodology that has the capability of capturing time-related fluctuations and unpredictable expert judgment. A combination of DIFSS and WASPAS approach would offer a good solution to enhance the precision and uniformity of renewable energy assessment. The rationale behind this study is thus to assist in making more credible, transparent and informed decision-making regarding the planning of renewable energy in uncertain and changing conditions.
The Table 2 shows the correlation between the research gaps that are found and the contributions made by this study. This mapping indicates that the proposed Dynamic Intuitionistic Fuzzy Soft Set-based WASPAS (DIFSS-WASPAS) focuses on the challenges that are evident in current MCDM methodologies of planning renewable energy.

Table 2: Mapping of Research Gaps to Study Contributions

Identified Research Gap	Corresponding Contribution of the Study
Most existing MCDM models for renewable energy selection assume a static decision environment and fail to accommodate changes over time.	Developed a Dynamic Intuitionistic Fuzzy Soft Set (DIFSS)–based framework capable of representing temporal variations in renewable energy decision-making.
Existing fuzzy and intuitionistic fuzzy approaches effectively represent uncertainty but lack mechanisms for parameterized uncertainty handling.	Introduced parameterization within the DIFSS framework to better capture multi-perspective and parameter-dependent uncertainty in decision processes.
Integration of soft set theory and intuitionistic fuzzy environments in a dynamic context remains limited in prior research.	Formulated a dynamic hybrid structure combining intuitionistic fuzzy theory with soft set parameterization, enabling improved uncertainty and dynamism representation.
Existing studies have rarely combined hybrid aggregation techniques such as WASPAS with DIFSS models for renewable energy planning.	Integrated the WASPAS hybrid aggregation method with the DIFSS framework, improving the stability, reliability, and accuracy of ranking renewable energy alternatives.
Current renewable energy decision models lack adaptability to dynamic criteria and evolving expert judgments in real-world settings.	Designed a dynamic evaluation mechanism within the proposed framework to incorporate expert feedback and temporally changing decision attributes for realistic energy planning.
Conventional decision models are not rigorously validated under uncertainty and time-varying conditions.	Conducted a renewable energy case study in Pakistan to empirically validate the model's capacity to produce consistent, robust, and interpretable results under uncertain dynamic conditions.

The rest of this paper is structured in the following way. Section 2 introduces the basic notions of soft sets, fuzzy soft sets as well as dynamic soft sets. Section 3 presents the suggested DIFSS model and explains its principal characteristics. The WASPAS approach with the associated theoretical outcomes is introduced in section 4. Section 5 represents the full procedure of the proposed DIFSS-WASPAS framework, whereas Section 6 represents the respective algorithm. Section 7 gives the case study and implementation of renewable energy evaluation. Section 8 gives the sensitivity analysis, results and discussion in Section 9. Lastly, a conclusion is given at the end of the paper and a way forward in future research is pointed out.

2. Preliminaries

This section presents several basic concepts related to fuzzy sets, intuitionistic fuzzy sets, soft sets, and their extensions which are used throughout the paper.

Definition 2.1. *A* ***fuzzy set****, introduced by Zadeh [28], is a set in which each element has a degree of* *membership ranging from* 0 *to* 1*. Let* U *be a universe of discourse. A fuzzy set* $\widetilde{A}$ *in* U *is defined as*

$$\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) \mid x \in U,\ \mu_{\widetilde{A}}(x) \in [0,1]\},$$

where $\mu_{\widetilde{A}} : U \to [0,1]$ *denotes the membership function of* $\widetilde{A}$*.*

Definition 2.2. *An* ***intuitionistic fuzzy set****, introduced by Atanassov [2], extends fuzzy sets by incorporating both membership and non-membership degrees. An intuitionistic fuzzy set* A *in a universe* U *is defined as*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in U\},$$

where $\mu_A(x)$ *and* $\nu_A(x)$ *represent the membership and non-membership degrees of* x*, respectively, satisfying*

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

The hesitation degree is defined as $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$.

Definition 2.3. *A* ***soft set****, introduced by Molodtsov [12], is a parameterized family of subsets of a universe. Let* U *be a universe and* A *be a set of parameters. A soft set* (F, A) *over* U *is defined as*

$$(F, A) = \{(a, F(a)) \mid a \in A,\ F(a) \subseteq U\},$$

where F *is a mapping*

$$F : A \to \mathcal{P}(U),$$

and $\mathcal{P}(U)$ *denotes the power set of* U*.*

Definition 2.4. *A* ***dynamic soft set*** *[16] is an extension of a soft set in which parameters or their corresponding mappings may change over time. It can be represented as*

$$\{(F_t, A_t) \mid t = 1, 2, \ldots, T\},$$

where t *represents the time or decision stage and* T *denotes the total number of stages.*

Definition 2.5. *An* ***intuitionistic fuzzy soft set*** *[9] combines the concepts of intuitionistic fuzzy sets and soft sets. Let* U *be a universe and* A *a set of parameters. An IFSS* (F, A) *over* U *is defined as*

$$(F, A) = \{(a, F(a)) \mid a \in A\},$$

where $F : A \to$ *IFS*(U)*. For each* $a \in A$ *and* $x \in U$*, the mapping assigns* $F(a)(x) = (\mu_{F(a)}(x), \nu_{F(a)}(x))$*, such that* $0 \leq \mu_{F(a)}(x) + \nu_{F(a)}(x) \leq 1$*. The hesitation degree is* $\pi_{F(a)}(x) = 1 - \mu_{F(a)}(x) - \nu_{F(a)}(x)$*.*

3. Dynamic Intuitionistic Fuzzy Soft Sets (DIFSS)

In this section, we formalize the concept of DIFSS and develop a complete suite of basic set-theoretical notions and operations that will be used in the subsequent DIFSS–WASPAS methodology. Throughout, U denotes the universe of alternatives (e.g., renewable options), A the set of parameters/criteria, and $T = \{t_1, \ldots, t_K\}$ the discrete time index set.

3.1 Definition and basic notation

Recall that an intuitionistic fuzzy soft set over (U, A) is a mapping

$$F : A \to \mathrm{IFS}(U),$$

where $\mathrm{IFS}(U)$ denotes the family of intuitionistic fuzzy sets on U. For each parameter $A_j \in C$, the image $F(A_j)$ is an intuitionistic fuzzy set on U and can be written as

$$F(A_j) = \{\langle x, \mu_{F(A_j)}(x), \nu_{F(A_j)}(x)\rangle \mid x \in U\},$$

with $\mu_{F(A_j)}(x), \nu_{F(A_j)}(x) \in [0, 1]$ and $\mu_{F(A_j)}(x) + \nu_{F(A_j)}(x) \leq 1$.

Definition 3.1. *A **DIFSS** over (U, A, T) is a family*

$$\mathcal{F} = \{(F_t, A_t) \mid t \in T\},$$

where for each $t \in T$, $A_t \subseteq A$ and $F_t : A_t \to \mathrm{IFS}(U)$ is an intuitionistic fuzzy soft set at time t. Equivalently, a DIFSS can be represented as a mapping $\mathcal{F} : A \times T \to \mathrm{IFS}(U)$, such that for each $(A_j, t_k) \in A \times T$,

$$\mathcal{F}(A_j, t_k) = \{\langle x, \mu_{jk}(x), \nu_{jk}(x)\rangle \mid x \in U\},$$

with $\mu_{jk}(x) + \nu_{jk}(x) \leq 1$ and hesitation degree $\pi_{jk}(x) = 1 - \mu_{jk}(x) - \nu_{jk}(x) \geq 0$.

In the decision-making context, we often denote

$$\mu_{ijk} = \mu_{jk}(L_i), \quad \nu_{ijk} = \nu_{jk}(L_i), \quad \pi_{ijk} = 1 - \mu_{ijk} - \nu_{ijk},$$

where $L_i \in U$ is the i-th alternative and (A_j, t_k) is a criterion time pair.

Example 3.1. *Let $U = \{L_1$: Solar$, L_2$: Wind$, L_3$: Hydro$\}$, $A = \{A_7$: Generation$\}$, and $T = \{2021, 2023\}$. A DIFSS $\mathcal{F}$ may assign for $(A_7, 2021)$:*

$$\mathcal{F}(A_7, 2021) = \{\langle L_1, 0.6, 0.3\rangle, \langle L_2, 0.8, 0.15\rangle, \langle L_3, 0.85, 0.10\rangle\},$$

and for $(A_7, 2023)$:

$$\mathcal{F}(A_7, 2023) = \{\langle L_1, 0.8, 0.15\rangle, \langle L_2, 0.85, 0.10\rangle, \langle L_3, 0.85, 0.10\rangle\},$$

reflecting temporal improvement in the intuitionistic evaluation of solar generation.

Definition 3.2. *Let $\mathcal{F} = \{(F_t, A_t) \mid t \in T\}$ and $\mathcal{G} = \{(G_t, D_t) \mid t \in T\}$ be two DIFSS over (U, A, T). We say that $\mathcal{F}$ is a **DIFSS subset** of $\mathcal{G}$, written $\mathcal{F} \subseteq \mathcal{G}$, if for every $t \in T$:*

1. $A_t \subseteq D_t$.
2. *For each $A_j \in A_t$ and each $x \in U$,*

$$\mu_{F_t(A_j)}(x) \leq \mu_{G_t(A_j)}(x), \quad \nu_{F_t(A_j)}(x) \geq \nu_{G_t(A_j)}(x).$$

Definition 3.3. *Two DIFSS $\mathcal{F}$ and $\mathcal{G}$ are **equal**, written $\mathcal{F} = \mathcal{G}$, if $\mathcal{F} \subseteq \mathcal{G}$ and $\mathcal{G} \subseteq \mathcal{F}$.*

Definition 3.4. *Let $\mathcal{F} = \{(F_t, A_t) \mid t \in T\}$ be a DIFSS over (U, A, T). The **complement** of $\mathcal{F}$, denoted $\mathcal{F}^c$, is the DIFSS $\mathcal{F}^c = \{(F_t^c, A_t) \mid t \in T\}$ such that for each $t \in T$, each $A_j \in C_t$ and each $x \in U$,*

$$\mu_{F_t^c(A_j)}(x) = \nu_{F_t(A_j)}(x), \quad \nu_{F_t^c(A_j)}(x) = \mu_{F_t(A_j)}(x).$$

The hesitation degree at (x, A_j, t) is preserved: $\pi_{F_t^c(A_j)}(x) = \pi_{F_t(A_j)}(x)$.

Proposition 3.1. *For any DIFSS $\mathcal{F}$ over (U, A, T):*

1. $(\mathcal{F}^c)^c = \mathcal{F}$.
2. *If $\mathcal{F} \subseteq \mathcal{G}$, then $\mathcal{G}^c \subseteq \mathcal{F}^c$.*

Proof. At each fixed (A_j, t), the identities follow directly from the corresponding properties of intuitionistic fuzzy complements, extended parameter-wise and time-wise. □

Definition 3.5. *Let $\mathcal{F} = \{(F_t, A_t)\}$ and $\mathcal{G} = \{(G_t, D_t)\}$ be two DIFSS over (U, A, T). The **union** $\mathcal{H} = \mathcal{F} \cup \mathcal{G}$ is defined as $\mathcal{H} = \{(H_t, E_t) \mid t \in T\}$ where*

1. $E_t = A_t \cup D_t$.
2. *For each $A_j \in E_t$ and $x \in U$,*

$$\mu_{H_t(A_j)}(x) = \begin{cases} \max\{\mu_{F_t(A_j)}(x), \mu_{G_t(A_j)}(x)\}, & A_j \in A_t \cap D_t, \\ \mu_{F_t(A_j)}(x), & A_j \in A_t \setminus D_t, \\ \mu_{G_t(A_j)}(x), & A_j \in D_t \setminus A_t, \end{cases}$$

$$\nu_{H_t(A_j)}(x) = \begin{cases} \min\{\nu_{F_t(A_j)}(x), \nu_{G_t(A_j)}(x)\}, & A_j \in A_t \cap D_t, \\ \nu_{F_t(A_j)}(x), & A_j \in A_t \setminus D_t, \\ \nu_{G_t(A_j)}(x), & A_j \in D_t \setminus A_t. \end{cases}$$

Definition 3.6. *Let $\mathcal{F}$ and $\mathcal{G}$ be as above. The **intersection** $\mathcal{H} = \mathcal{F} \cap \mathcal{G}$ is defined as $\mathcal{H} = \{(H_t, E_t) \mid t \in T\}$ where*

1. $E_t = A_t \cap D_t$.
2. *For each $A_j \in E_t$ and $x \in U$,*

$$\mu_{H_t(A_j)}(x) = \min\{\mu_{F_t(A_j)}(x), \mu_{G_t(A_j)}(x)\},$$

$$\nu_{H_t(A_j)}(x) = \max\{\nu_{F_t(A_j)}(x), \nu_{G_t(A_j)}(x)\}.$$

In both operations, the hesitation degree is implicitly updated as $\pi(x) = 1 - \mu(x) - \nu(x)$ and therefore remains compatible with intuitionistic fuzzy constraints.

Proposition 3.2. *For any DIFSS $\mathcal{F}, \mathcal{G}, \mathcal{H}$ over (U, A, T):*

1. Commutativity: $\mathcal{F} \cup \mathcal{G} = \mathcal{G} \cup \mathcal{F}$, $\mathcal{F} \cap \mathcal{G} = \mathcal{G} \cap \mathcal{F}$.
2. Associativity: $(\mathcal{F} \cup \mathcal{G}) \cup \mathcal{H} = \mathcal{F} \cup (\mathcal{G} \cup \mathcal{H})$, $(\mathcal{F} \cap \mathcal{G}) \cap \mathcal{H} = \mathcal{F} \cap (\mathcal{G} \cap \mathcal{H})$.
3. Idempotency: $\mathcal{F} \cup \mathcal{F} = \mathcal{F}$, $\mathcal{F} \cap \mathcal{F} = \mathcal{F}$.
4. Absorption: $\mathcal{F} \cup (\mathcal{F} \cap \mathcal{G}) = \mathcal{F}$, $\mathcal{F} \cap (\mathcal{F} \cup \mathcal{G}) = \mathcal{F}$.

Proof. Each property follows from the corresponding properties of max–min based union and intersection of intuitionistic fuzzy sets, applied pointwise for each parameter A_j and each time t. □

Proposition 3.3. *For any DIFSS $\mathcal{F}, \mathcal{G}$ over (U, A, T),*

$$(\mathcal{F} \cup \mathcal{G})^c = \mathcal{F}^c \cap \mathcal{G}^c, \quad (\mathcal{F} \cap \mathcal{G})^c = \mathcal{F}^c \cup \mathcal{G}^c.$$

Proof. At each fixed (A_j, t), the definitions reduce to standard De Morgan identities for intuitionistic fuzzy sets; parameterization and temporal indexing do not affect the algebraic structure. □

Definition 3.7. *The **DIFSS null set** $\mathcal{O}$ over (U, A, T) is defined by $\mathcal{O} = \{(O_t, A) \mid t \in T\}$, where for each $A_j \in A$ and each $x \in U$,*

$$\mu_{O_t(A_j)}(x) = 0, \quad \nu_{O_t(A_j)}(x) = 0, \quad \pi_{O_t(A_j)}(x) = 1.$$

Definition 3.8. *The **DIFSS universal set** $\mathcal{U}$ over (U, A, T) is defined by*

$$\mathcal{U} = \{(U_t, A) \mid t \in T\},$$

where for each $A_j \in A$ and each $x \in U$,

$$\mu_{U_t(A_j)}(x) = 1, \quad \nu_{U_t(A_j)}(x) = 0, \quad \pi_{U_t(A_j)}(x) = 0.$$

Proposition 3.4. *For any DIFSS $\mathcal{F}$ over (U, C, T),*

1. $\mathcal{F} \cup \mathcal{O} = \mathcal{F}$, $\mathcal{F} \cap \mathcal{U} = \mathcal{F}$.
2. $\mathcal{F} \cup \mathcal{U} = \mathcal{U}$, $\mathcal{F} \cap \mathcal{O} = \mathcal{O}$.
3. $\mathcal{O}^c = \mathcal{U}$, $\mathcal{U}^c = \mathcal{O}$.

Proof. Direct from the max–min definitions of union, intersection and complement and the extremal membership/non-membership values of $\mathcal{O}$ and $\mathcal{U}$. □

3.2 Dynamic restriction and projection

Dynamic decision environments often involve situations where only a subset of parameters or alternatives is relevant at a particular time stage. Therefore, the concepts of dynamic restriction and projection are introduced to extract and analyze the corresponding parts of a dynamic intuitionistic fuzzy soft set with respect to selected parameters or subsets of the universe.

Definition 3.9. *Let $\mathcal{F} = \{(F_t, A_t)\}$ be a DIFSS over (U, A, T) and let $A' \subseteq A$. The **parameter restriction** of $\mathcal{F}$ to A', denoted $\mathcal{F}|_{A'}$, is the DIFSS*

$$\mathcal{F}|_{A'} = \{(F_t|_{A'}, A_t \cap A') \mid t \in T\},$$

where $F_t|_{A'}$ is the restriction of F_t to the reduced parameter set $A_t \cap A'$.

Definition 3.10. *Let $T' \subseteq T$. The **temporal restriction** of $\mathcal{F}$ to T', denoted $\mathcal{F}|_{T'}$, is the family*

$$\mathcal{F}|_{T'} = \{(F_t, A_t) \mid t \in T'\},$$

which is again a DIFSS over (U, A, T').

These restriction operators are compatible with the exponential recency aggregation used later: if the DIFSS is restricted to a shorter horizon $T' \subseteq T$, the temporal weights α_k in the aggregation formula $x_{ij} = \sum_{k=1}^{K} \alpha_k s_{ijk}$ are simply recomputed over the new index set T'.

3.3 Dynamic intuitionistic fuzzy soft relations

In this subsection, the concept of dynamic intuitionistic fuzzy soft relations is introduced to describe relationships between elements of two universes under parameters and time stages.

Definition 3.11. *Let U and V be two universes, and let A and D be parameter sets on U and V, respectively. A* ***dynamic intuitionistic fuzzy soft relation*** *from (U, A, T) to (V, D, T) is a mapping*

$$\mathcal{R} : A \times D \times T \to \mathrm{IFS}(U \times V),$$

where for each (A_j, D_ℓ, t_k), the image $\mathcal{R}(A_j, D_\ell, t_k)$ is an intuitionistic fuzzy relation on $U \times V$.

The basic operations (inverse, composition, restriction) of such relations can be defined in the usual way, parameter-wise and time-wise, using the intuitionistic fuzzy max–min composition. Although dynamic relations are not directly exploited in the present DIFSS WASPAS model, they provide a natural bridge to future work on dynamic intuitionistic fuzzy soft graphs and networked energy systems.

4. WASPAS Method

The Weighted Aggregated Sum Product Assessment (WASPAS) method is a multi-criteria decision-making technique that integrates the weighted sum model (WSM) and the weighted product model (WPM). By combining these two approaches, WASPAS provides a more reliable evaluation of alternatives based on multiple criteria.

Definition 4.1. *The WASPAS method determines the overall performance score of each alternative by combining the weighted sum and weighted product models. The performance score of alternative i is given by*

$$Q_i = \lambda \sum_{j=1}^{n} w_j x_{ij} + (1-\lambda) \prod_{j=1}^{n} x_{ij}^{w_j},$$

where w_j is the weight of criterion j, x_{ij} is the normalized performance value of alternative i under criterion j, and $\lambda \in [0,1]$ is a parameter that controls the relative contribution of the weighted sum and weighted product models.

Example 4.1. *Consider two alternatives, solar ($i = 1$) and wind ($i = 2$), evaluated with respect to two benefit criteria having weights*

$$w_1 = 0.4, \qquad w_2 = 0.6.$$

Suppose the normalized performance values for solar are

$$x_{11} = 0.8, \qquad x_{12} = 0.7.$$

Let $\lambda = 0.5$. Then the WASPAS score for solar is

$$Q_1 = 0.5\big(0.4 \times 0.8 + 0.6 \times 0.7\big) + 0.5\big(0.8^{0.4} \times 0.7^{0.6}\big).$$

Theorem 4.1. *Let $0 \le x_{ij} \le 1$ and $w_j \ge 0$ with $\sum_{j=1}^{n} w_j = 1$. Then the weighted product component of the WASPAS operator*

$$Q_i^{(2)} = \prod_{j=1}^{n} x_{ij}^{w_j}$$

satisfies

$$0 \le Q_i^{(2)} \le 1.$$

Proof. Since $0 \leq x_{ij} \leq 1$ and $w_j \geq 0$, we have

$$0 \leq x_{ij}^{w_j} \leq 1$$

for each $j = 1, 2, \ldots, n$.

Because the product of numbers in the interval $[0, 1]$ also lies in $[0, 1]$, it follows that

$$0 \leq \prod_{j=1}^{n} x_{ij}^{w_j} \leq 1.$$

Hence

$$0 \leq Q_i^{(2)} \leq 1.$$

This completes the proof. □

Theorem 4.2. *Let $0 \leq x_{ij} \leq 1$ and $w_j \geq 0$ with $\sum_{j=1}^{n} w_j = 1$. Then the WASPAS score*

$$Q_i = \lambda \sum_{j=1}^{n} w_j x_{ij} + (1 - \lambda) \prod_{j=1}^{n} x_{ij}^{w_j}$$

satisfies

$$0 \leq Q_i \leq 1.$$

Proof. Since $0 \leq x_{ij} \leq 1$ and $w_j \geq 0$, we have

$$0 \leq \sum_{j=1}^{n} w_j x_{ij} \leq 1$$

because $\sum_{j=1}^{n} w_j = 1$.

Similarly, for the weighted product term,

$$0 \leq \prod_{j=1}^{n} x_{ij}^{w_j} \leq 1$$

since each $x_{ij}^{w_j} \in [0, 1]$.

Because $\lambda \in [0, 1]$, the convex combination

$$Q_i = \lambda Q_i^{(1)} + (1 - \lambda) Q_i^{(2)}$$

also lies in $[0, 1]$. Hence $0 \leq Q_i \leq 1$. □

Theorem 4.3. *Let x_{ij} and y_{ij} be two normalized decision matrices such that*

$$x_{ij} \leq y_{ij} \quad \textit{for all } j.$$

Then the corresponding WASPAS scores satisfy

$$Q_i(x_{ij}) \leq Q_i(y_{ij}).$$

Proof. Since $x_{ij} \le y_{ij}$ for all j, we have

$$\sum_{j=1}^{n} w_j x_{ij} \le \sum_{j=1}^{n} w_j y_{ij}.$$

Similarly, because the function x^w is increasing for $x \in [0,1]$ and $w \ge 0$, it follows that

$$\prod_{j=1}^{n} x_{ij}^{w_j} \le \prod_{j=1}^{n} y_{ij}^{w_j}.$$

Combining these inequalities and using $\lambda \in [0,1]$, we obtain

$$Q_i(x_{ij}) \le Q_i(y_{ij}).$$

Thus, the WASPAS operator is monotonic. □

Theorem 4.4. *If all normalized values are equal, i.e.,*

$$x_{i1} = x_{i2} = \cdots = x_{in} = c, \quad c \in [0,1],$$

then the WASPAS score satisfies

$$Q_i = c.$$

Proof. If $x_{ij} = c$ for all j, then

$$\sum_{j=1}^{n} w_j x_{ij} = c \sum_{j=1}^{n} w_j = c.$$

Also,

$$\prod_{j=1}^{n} x_{ij}^{w_j} = \prod_{j=1}^{n} c^{w_j} = c^{\sum_{j=1}^{n} w_j} = c.$$

Therefore,

$$Q_i = \lambda c + (1-\lambda)c = c.$$

Hence the WASPAS operator is idempotent. □

5. Methodology

This study proposes a DIFSS–WASPAS framework for ranking renewable energy alternatives under time-varying and uncertain expert evaluations. The proposed methodology integrates dynamic intuitionistic fuzzy soft modelling with the WASPAS multi-criteria decision-making approach.
The procedure consists of four main stages: (i) modelling dynamic intuitionistic fuzzy information, (ii) constructing the aggregated decision matrix, (iii) applying the WASPAS aggregation mechanism, and (iv) ranking the alternatives.

5.1 Problem formulation and notation

Let $A = \{A_1, \ldots, A_m\}$ denote the set of renewable energy alternatives and $C = \{C_1, \ldots, C_n\}$ the set of evaluation criteria, classified as benefit or cost type. Expert assessments are collected over discrete time periods $T = \{t_1, \ldots, t_K\}$ in order to capture temporal variations in technological, economic, and environmental conditions.
Within the intuitionistic fuzzy framework, the evaluation of alternative A_i with respect to criterion C_j at time t_k is represented by an intuitionistic fuzzy pair $\tilde{x}_{ijk} = (\mu_{ijk}, \nu_{ijk})$, where μ_{ijk} and ν_{ijk} denote the membership and non-membership degrees, respectively. The hesitation degree is defined as $\pi_{ijk} = 1 - \mu_{ijk} - \nu_{ijk}$, subject to the constraint $0 \le \mu_{ijk} + \nu_{ijk} \le 1$.

5.2 *Dynamic intuitionistic fuzzy soft set modelling*

To represent parameterised and time-dependent evaluations, the information is organized as a DIFSS over the universe A with parameter set C and time index T. Formally, a DIFSS is defined as the mapping $F : C \times T \to \mathrm{IFS}(A)$, where $F(C_j, t_k)$ provides the collection of intuitionistic fuzzy evaluations $\{\tilde{x}_{ijk}\}_{i=1}^{m}$ for all alternatives under criterion C_j at time t_k.
In practical applications, experts often provide linguistic assessments such as “low”, “medium”, or “high”. These linguistic terms are converted into corresponding intuitionistic fuzzy pairs (μ, ν) using a predefined linguistic scale. When multiple experts participate in the evaluation process, their individual assessments for a given (i, j, k) are aggregated through an intuitionistic fuzzy weighted averaging operator to obtain a consensus intuitionistic fuzzy pair $\tilde{x}_{ijk}$.
Figure 1 illustrates the overall workflow of the proposed DIFSS–WASPAS decision-making framework.

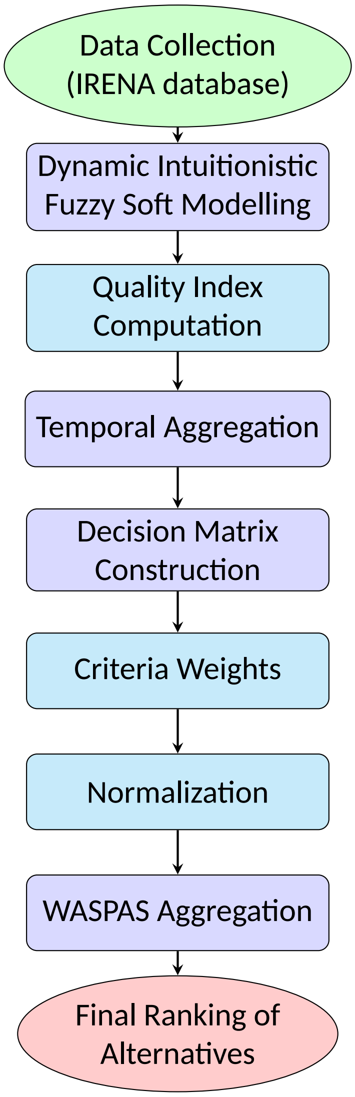

Figure 1: Workflow of the proposed DIFSS–WASPAS decision framework.

5.3 *Quality index and temporal aggregation*

In order to apply the WASPAS decision framework, intuitionistic fuzzy evaluations must be transformed into scalar performance scores. For this purpose, a hesitation-preserving quality index is introduced to convert each intuitionistic fuzzy evaluation into a normalized numerical value. Each intuitionistic fuzzy evaluation $\tilde{x}_{ijk} = (\mu_{ijk}, \nu_{ijk})$ with hesitation degree $\pi_{ijk} = 1 - \mu_{ijk} - \nu_{ijk}$ is transformed into a normalized score $s_{ijk} \in [0, 1]$ defined as

$$s_{ijk} = \frac{\mu_{ijk}}{\mu_{ijk} + \nu_{ijk} + 0.01} + \frac{\pi_{ijk}}{2(\mu_{ijk} + \nu_{ijk} + 0.01)}. \tag{1}$$

The first term reflects the relative support for the alternative, while the second term incorporates the hesitation degree, allowing uncertain information to partially contribute to the final score. Because evaluations are observed across multiple time periods, a temporal aggregation mechanism is required to obtain a single representative score for each alternative–criterion pair. An exponential recency weighting scheme is adopted:

$$x_{ij} = \sum_{k=1}^{K} \alpha_k s_{ijk}, \qquad \alpha_k = \frac{\lambda^{K-k}}{\sum_{l=1}^{K} \lambda^{K-l}}, \tag{2}$$

where $\lambda \in [0, 1]$ controls the emphasis placed on recent observations. In this study, $\lambda = 0.85$ is used, reflecting a moderate preference for recent information while still retaining the influence of historical evaluations. For $K = 3$ periods (2021–2023), the resulting weights are

$$\alpha_{2021} = 0.24, \quad \alpha_{2022} = 0.29, \quad \alpha_{2023} = 0.47.$$

Example 5.1. *Consider solar energy under the generation criterion with linguistic ratings* $\{H, H, VH\}$ *over three periods. These are converted into scores* $\{0.74, 0.74, 0.92\}$*. The aggregated value is*

$$x_{11} = 0.24(0.74) + 0.29(0.74) + 0.47(0.92) = 0.80.$$

5.3.1 *Properties of the quality index*

The proposed scalarization satisfies several desirable mathematical properties.

Theorem 5.1. *Let* s_{ijk} *be defined by Eq. (3) for any intuitionistic fuzzy evaluation* (μ_{ijk}, ν_{ijk}) *with* $0 \leq \mu_{ijk}, \nu_{ijk} \leq 1$ *and* $\mu_{ijk} + \nu_{ijk} \leq 1$*. Then the following properties hold.*

1. ***Boundedness.*** $0 \leq s_{ijk} \leq 1$.
2. ***Monotonicity in membership.*** *If* μ_{ijk} *increases while* ν_{ijk} *does not increase, then* s_{ijk} *does not decrease.*
3. ***Monotonicity in non-membership.*** *If* ν_{ijk} *increases while* μ_{ijk} *does not increase, then* s_{ijk} *does not increase.*
4. ***Reduction under zero hesitation.*** *If* $\pi_{ijk} = 0$ *(i.e.,* $\mu_{ijk} + \nu_{ijk} = 1$*), then* $s_{ijk} = \frac{\mu_{ijk}}{1.01}$.

Proof. The results follow directly from the definition of the score function and the admissible constraints of intuitionistic fuzzy sets. □

5.3.2 Limiting cases of the DIFSS–WASPAS framework

The proposed DIFSS–WASPAS framework generalizes several existing decision-making configurations. In particular, it reduces to well-known special cases under certain parameter settings.

- **Case 1: Static IFSS–WASPAS (no temporal dynamics).** If only a single time period is considered ($K = 1$), temporal aggregation is not required and the aggregated value reduces to $x_{ij} = s_{ij1}$. Consequently, the framework becomes a static intuitionistic fuzzy soft set (IFSS)–based WASPAS model without temporal dynamics.

- **Case 2: Dynamic fuzzy soft set WASPAS (no hesitation).** If the hesitation degree vanishes for all evaluations, $\pi_{ijk} = 0$, then $\mu_{ijk} + \nu_{ijk} = 1$ and the intuitionistic representation collapses to a classical fuzzy evaluation. In this situation, the quality index depends solely on the membership degree μ_{ijk}, and the proposed framework effectively reduces to a dynamic fuzzy soft set–based WASPAS model.

- **Case 3: Pure WSM or pure WPM aggregation.** In the WASPAS method, the overall score of alternative A_i is $Q_i = \lambda Q_i^{(1)} + (1 - \lambda)Q_i^{(2)}$. where $Q_i^{(1)}$ and $Q_i^{(2)}$ denote the weighted sum model (WSM) and weighted product model (WPM) scores, respectively. If $\lambda = 1$, the framework reduces to the additive WSM: $Q_i = Q_i^{(1)}$. If $\lambda = 0$, it reduces to the multiplicative WPM: $Q_i = Q_i^{(2)}$.
  These limiting cases illustrate that the proposed DIFSS–WASPAS framework unifies several existing approaches, including static IFSS–WASPAS, dynamic fuzzy soft WASPAS, and the classical WSM/WPM decision models.

5.3.3 Conceptual comparison with existing intuitionistic fuzzy score functions

Traditional score and accuracy functions for intuitionistic fuzzy sets typically transform (μ, ν, π) into a single scalar value by emphasizing membership while penalizing non-membership, often treating hesitation implicitly. In contrast, the proposed quality index explicitly incorporates the hesitation degree π as an additional positive component. This design has two important implications:

1. Evaluations with identical membership values but different hesitation degrees do not collapse to the same score. Instead, the index preserves the uncertainty information represented by π.

2. The temporally aggregated scores are obtained using exponential recency weighting. As a result, time-varying hesitation levels directly influence the aggregated decision matrix that serves as input to the WASPAS ranking procedure.

This property is particularly valuable in dynamic expert-based energy planning problems, where both the degree of support and the degree of uncertainty may evolve over time.

5.4 Determination of criteria weights

The relative importance of the evaluation criteria is determined based on expert judgments. Each expert assigns an importance score to every criterion using a numerical scale ranging from 0 to 10, where larger values indicate higher importance. Let s_j denote the average importance score assigned to criterion C_j after aggregating the responses of all experts. The normalized weight of criterion C_j is then computed as

$$w_j = \frac{s_j}{\sum_{l=1}^{n} s_l}, \quad j = 1, \dots, n,$$

which guarantees that $\sum_{j=1}^{n} w_j = 1$. The resulting weight vector $w = (w_1, w_2, \ldots, w_n)$ is subsequently used in the WASPAS aggregation procedure.

5.5 *WASPAS aggregation procedure*

The WASPAS (Weighted Aggregated Sum Product Assessment) method integrates the weighted sum model (WSM) and the weighted product model (WPM) to produce reliable rankings of decision alternatives [4, 29]. In this study, the method is applied to rank renewable energy alternatives $\{\text{Solar}, \text{Wind}, \text{Hydro}, \text{Biomass}\}$. Given the aggregated decision matrix $X = [x_{ij}]$ and the weight vector w, the procedure consists of the following steps.

Step 1: Normalization. To eliminate the influence of measurement scales, the decision matrix is normalized. For benefit-type criteria, normalization is performed as

$$r_{ij} = \frac{x_{ij}}{\max_i x_{ij}},$$

while for cost-type criteria it is computed as $r_{ij} = \frac{\min_i x_{ij}}{x_{ij}}$. This yields the normalized decision matrix $R = [r_{ij}]$.

Step 2: Weighted sum model (WSM). The additive performance score of alternative A_i is calculated as

$$Q_i^{(1)} = \sum_{j=1}^{n} w_j r_{ij}.$$

Step 3: Weighted product model (WPM). The multiplicative performance score of alternative A_i is obtained by

$$Q_i^{(2)} = \prod_{j=1}^{n} r_{ij}^{w_j}.$$

Step 4: WASPAS combined score. The final WASPAS score is computed by combining the WSM and WPM scores through a balancing parameter $\lambda \in [0, 1]$:

$$Q_i = \lambda Q_i^{(1)} + (1 - \lambda) Q_i^{(2)}.$$

The parameter λ controls the relative contribution of the additive and multiplicative aggregation mechanisms. In many practical applications, $\lambda = 0.5$ is adopted in order to assign equal importance to both models. Sensitivity analysis with respect to λ can be performed to examine the stability of the resulting rankings.

5.6 *Ranking and analysis*

The alternatives are ranked according to the descending order of the WASPAS score Q_i. The alternative with the highest score is considered the most suitable renewable energy option under the dynamic intuitionistic fuzzy soft set environment. To evaluate the robustness of the proposed DIFSS–WASPAS framework, additional analyses are conducted. These include sensitivity analysis with respect to the parameter λ and the criterion weights, as well as comparative experiments with established multi-criteria decision-making methods such as TOPSIS and VIKOR.

6. Algorithm

This section presents the procedure for ranking renewable energy alternatives under dynamic intuitionistic fuzzy soft information. The steps of the proposed DIFSS–WASPAS decision-making framework are summarized in Algorithm 1.

7. Case Study: Renewable Energy Planning in Pakistan

In order to prove the relevance and practicality of the suggested DIFSS-WASPAS framework, a renewable energy planning issue in Pakistan is taken into account. The aim will be to assess and rank the various renewable energy options on a number of economic, environmental and technical factors and include time data over a span of years.
During Step 1 (Problem Definition and Criteria Selection), four alternative renewable energy options are considered: $A = \{A_1$: Solar, A_2 : Wind, A_3 : Hydro, A_4 : Bioenergy$\}$. These options are evaluated using eight criteria of cost, environmental impact and technical performance: $C = \{C_1$: LCOE, C_2 : CO_2 savings, C_3 : Land use, C_4 : Grid stability, C_5 : CAPEX, C_6 : OPEX, C_7 : Generation, C_8 : Capacity factor$\}$. The criteria weights w_j are located by the consultation of experts and meet the requirement of normalization $\sum w_j = 1$. Table 3 reflects the criteria, weights of the criteria and their classification as benefit or cost attributes.

Table 3: Evaluation criteria and expert weights ($\sum w_j = 1$)

Criteria	**Weight** w_j	**Type**
C1: LCOE	0.15	Cost
C2: CO_2 Savings	0.12	Benefit
C3: Land Use	0.10	Cost
C4: Grid Stability	0.13	Benefit
C5: CAPEX	0.15	Cost
C6: OPEX	0.10	Benefit
C7: Generation	0.20	Benefit
C8: Capacity Factor	0.15	Benefit

In Step 2 (DIFSS Data Collection, 2021–2023), evaluation data were collected over three years to capture temporal variations in renewable energy performance. The Dynamic Intuitionistic Fuzzy Soft Set (DIFSS) structure enables the integration of multi-period information through temporal weighting. Table 4 illustrates the evolution of the generation criterion (C_7) across the three years for each alternative, together with the corresponding dynamically aggregated score.

Table 4: DIFSS evolution for generation criterion (C_7) across three years

Alternative	2021 (s_{i71})	2022 (s_{i72})	2023 (s_{i73})	Dynamic score (x_{i7})
Solar	0.74	0.74	0.92	0.80
Wind	0.85	0.85	0.85	0.85
Hydro	0.85	0.85	0.85	0.85
Bioenergy	0.30	0.30	0.30	0.30
Temporal weights λ_k	0.24	0.29	0.47	1.00

Algorithm 1 DIFSS–WASPAS decision-making algorithm

Input: $A = \{A_1, A_2, \ldots, A_m\}$ (alternatives), $C = \{C_1, C_2, \ldots, C_n\}$ (criteria), $T = \{t_1, t_2, \ldots, t_K\}$ (time periods), $w = \{w_1, w_2, \ldots, w_n\}$ (weights), $\lambda \in [0, 1]$
Output: WASPAS scores and ranked alternatives

Step 1: Construct DIFSS decision matrix
for each criterion C_j **do**
  for each alternative A_i **do**
    for each time period t_k **do**
      Convert linguistic evaluation to $\tilde{x}_{ijk} = (\mu_{ijk}, \nu_{ijk})$
      Compute hesitation degree $\pi_{ijk} = 1 - \mu_{ijk} - \nu_{ijk}$
    end for
  end for
end for

Step 2: Compute quality index
for each (i, j, k) **do**
  $s_{ijk} = \frac{\mu_{ijk}}{\mu_{ijk}+\nu_{ijk}+0.01} + \frac{\pi_{ijk}}{2(\mu_{ijk}+\nu_{ijk}+0.01)}$
end for

Step 3: Temporal aggregation
Compute recency weights α_k
for each alternative A_i and criterion C_j **do**
  $x_{ij} = \sum_{k=1}^{K} \alpha_k s_{ijk}$
end for
Aggregated matrix $X = [x_{ij}]$

Step 4: Normalization
for each criterion C_j **do**
  if C_j is benefit **then**
    $$r_{ij} = \frac{x_{ij}}{\max_i x_{ij}}$$
  else
    $$r_{ij} = \frac{\min_i x_{ij}}{x_{ij}}$$
  end if
end for
$R = [r_{ij}]$

Step 5: Compute WASPAS scores
$Q_i^{(1)} = \sum_{j=1}^{n} w_j r_{ij}$
$Q_i^{(2)} = \prod_{j=1}^{n} r_{ij}^{w_j}$
$Q_i = \lambda Q_i^{(1)} + (1 - \lambda) Q_i^{(2)}$

Step 6: Rank alternatives
Sort alternatives in descending order of Q_i

The results indicate that solar energy exhibits significant improvement in generation capability, increasing from 0.74 in 2021 to 0.92 in 2023. However, the dynamic aggregation produces a more balanced long-term estimate of 0.80, avoiding the potential overestimation associated with using only the most recent year. Figure 2 illustrates how dynamic DIFSS aggregation ($\lambda = 0.85$) captures solar generation improvement from 0.74 (2021) to 0.92 (2023), producing a balanced score of 0.80 versus the static 2023 snapshot of 0.92.

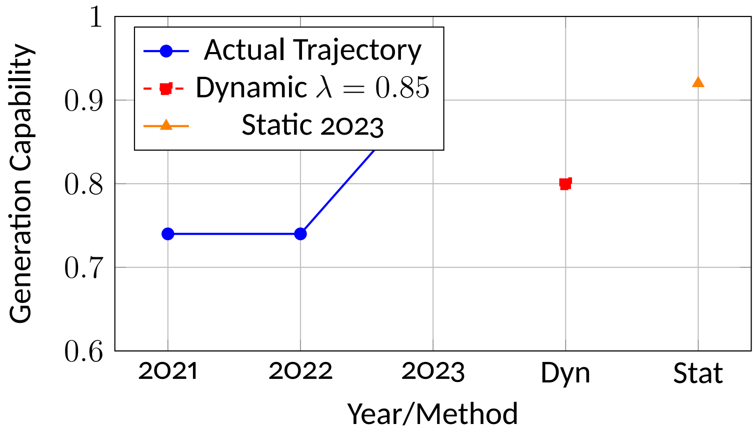

Figure 2: Temporal evolution of the solar generation criterion using DIFSS aggregation

Step 3 follows the computation of quality indices. The intuitionistic fuzzy evaluations are transformed into quantitative quality scores using the following scoring function:

$$s_{ijk} = \frac{\mu_{ijk}}{\mu_{ijk} + \nu_{ijk} + 0.01} + \frac{\pi_{ijk}}{2(\mu_{ijk} + \nu_{ijk} + 0.01)} \tag{3}$$

where μ_{ijk}, ν_{ijk}, and π_{ijk} represent the membership, non-membership, and hesitation degrees respectively. The resulting averaged quality scores for each alternative and criterion are presented in Table 5.

Table 5: Quality indices s_{ij} (average across 2021–2023)

	C1	C2	C3	C4	C5	C6	C7	C8
Solar	0.62	0.67	0.85	0.50	0.67	0.50	0.74	0.50
Wind	0.85	0.67	0.73	0.30	0.85	0.67	0.85	0.67
Hydro	0.67	0.85	0.30	0.85	0.50	0.30	0.85	0.85
Bioenergy	0.30	0.50	0.85	0.30	0.30	0.85	0.30	0.30

Step 4 evaluates the temporal aggregation of the multi-period evaluations. Temporal aggregation is performed using recency-based weights, where the aggregated score for each alternative and criterion is computed as $x_{ij} = \sum_{k=1}^{3} \lambda_k s_{ijk}$. Here, $\lambda_{2021} = 0.24$, $\lambda_{2022} = 0.29$, and $\lambda_{2023} = 0.47$ represent the temporal weights assigned to the three evaluation periods. These weights assign greater importance to more recent observations while still incorporating historical information, thereby capturing the dynamic performance of each alternative over time.
Step 5 performs the normalization of the aggregated decision matrix according to the WASPAS procedure. For benefit criteria, normalization is carried out using $r_{ij} = \frac{x_{ij}}{\max(x_j)}$, whereas cost criteria are normalized as $r_{ij} = \frac{\min(x_j)}{x_{ij}}$. The resulting normalized decision matrix is presented in Table 6.

Table 6: Normalized decision matrix

	C1	C2	C3	C4	C5	C6	C7	C8
Solar	0.73	0.79	0.35	0.59	0.75	0.59	0.87	0.59
Wind	1.00	0.79	0.41	0.35	1.00	0.79	1.00	0.79
Hydro	0.88	1.00	1.00	1.00	0.50	1.00	1.00	1.00
Bioenergy	0.44	0.59	0.35	0.35	0.38	0.35	0.35	0.35

In step 6, WASPAS Evaluation The final evaluation combines the weighted sum model (WSM) and the weighted product model (WPM):

$$Q_i^1 = \sum_{j=1}^{8} w_j r_{ij}, \tag{4}$$

$$Q_i^2 = \prod_{j=1}^{8} r_{ij}^{w_j}, \tag{5}$$

$$Q_i = 0.5(Q_i^1 + Q_i^2). \tag{6}$$

Table 7 presents the resulting scores and rankings of the alternatives.

Table 7: Final DIFSS–WASPAS rankings

Alternative	Q_i^1	Q_i^2	Q_i
Hydropower	0.952	0.941	**0.947 (1st)**
Wind	0.829	0.817	0.823 (2nd)
Solar	0.801	0.787	0.794 (3rd)
Bioenergy	0.315	0.309	0.312 (4th)

The ranking is maintained in all perturbation conditions, which proves the effectiveness and stability of the offered DIFSS-WASPAS framework. The last ranking of the alternatives is: Hydropower $>$ Wind $>$ Solar $>$ Bioenergy.

8. Sensitivity Analysis

In order to determine the strength and validity of the proposed DIFSS-WASPAS framework, a number of sensitivity and validation analyses are carried. These analyses are done to evaluate the stability of the obtained rankings to changes in the criteria weight, alteration in the WASPAS balancing parameter λ, and various scenarios of preference of decision-makers. Besides that, comparative validation with a proven method of multi-criteria decision-making and dynamic-static comparison is done to analyze the effectiveness of the proposed method further.

8.0.1 Weight Sensitivity

Sensitivity analysis is performed to ascertain the effect of criteria importance on the final ranking by introducing $\pm 40\%$ perturbation to the dominant criteria weights. The 8 values were obtained as reported in Table Q_i in each perturbation scenario. The findings indicate that the rankings of the alternatives are absolutely constant throughout the perturbations and a 100% rank correlation of alternatives to the base case. This fact implies that the moderate changes in the importance of criteria do not influence the decision results greatly, which proves the strength of the proposed framework.

Table 8: Weight Sensitivity: Perfect Rank Stability

Scenario	Hydro	Wind	Solar	Bioenergy
Base Case	0.947	0.823	0.794	0.312
LCOE $\pm 40\%$	0.943	0.827	0.798	0.310
Generation $\pm 40\%$	0.951	0.819	0.790	0.308
Grid Stability $\pm 40\%$	0.945	0.821	0.796	0.314
Rank Stability	**100%**	100%	100%	100%

8.0.2 WASPAS Parameter Sensitivity

The final scores $Q_i(\lambda)$ are calculated in order to explore the impact of the WASPAS balancing parameter on $\lambda \in \{0.2, 0.5, 0.8\}$. The parameter λ determines the relative value of the weight of the components of the Weighted Sum Model (WSM) and the weight of the components of the weighted product model (WPM). Particularly, $\lambda = 0.2$ lays stress on the multiplicative component of WPM, $\lambda = 0.8$ stresses the additive component of WSM and places the same value on both parts.

Table 9: Q_i values for different λ

Alternative	$Q_i(0.2)$		$Q_i(0.5)$		$Q_i(0.8)$	
	Value	Rank	Value	Rank	Value	Rank
Hydropower	0.944	1	0.947	1	0.949	1
Wind	0.821	2	0.823	2	0.825	2
Solar	0.792	3	0.794	3	0.796	3
Bioenergy	0.310	4	0.312	4	0.314	4

These changes are only slight variations in the Q_i absolute values with a variation in λ between 0.2 and 0.8 whereas the rank of alternatives does not alter. This shows that the DIFSS-WASPAS model is also not sensitive to the rational changes in the balance between the WSM and WPM components.

8.1 Scenario Analysis with Alternative Weight Vectors

Two further weighting scenarios are taken into account to investigate the effect of the preference of the various decision makers. **Scenario A (cost-focused):** emphasized the LCOE and CAPEX, and deemphasized the environmental criteria like CO_2 emissions and land use.
Scenario B (environment-focused): greater weight on CO_2 cutback and efficiency of land use, with economic cost criteria getting comparatively smaller weights.
The weights are re-normalized again in every instance to reduce the weight of $\sum_j w_j = 1$, and the DIFSS-WASPAS process is repeated in order to calculate the respective Q_i scores.

Table 10: Final Q_i under different weighting scenarios

Alternative	Base weights	Scenario A (cost)	Scenario B (env.)
Hydropower	0.947 (1st)	0.952 (1st)	0.941 (1st)
Wind	0.823 (2nd)	0.829 (2nd)	0.817 (2nd)
Solar	0.794 (3rd)	0.788 (3rd)	0.802 (3rd)
Bioenergy	0.312 (4th)	0.308 (4th)	0.315 (4th)

The ranking Hydro $>$ Wind $>$ Solar $>$ Bioenergy does not change in terms of the cost-focused and environment-focused situation. This implies that the suggested DIFSS-WASPAS paradigm ensures the consistent decision-making even in cases when decision-makers have alternative policy-related priorities.

8.2 *Comparative Validation*

In order to further justify the efficiency of the proposed model, the DIFSS-WASPAS results are also correlated with some well-established multi-criteria decision-making models, such as TOPSIS, VIKOR, and SAW based on the same eight evaluation criteria. Table 11 shows the comparative values and Spearman rank correlation coefficients.

Table 11: Benchmark Comparison (Spearman Rank Correlation)

Method	Hydro	Wind	Solar	Bioenergy	ρ
DIFSS-WASPAS	0.947	0.823	0.794	0.312	-
TOPSIS	0.923	0.812	0.785	0.298	0.95
VIKOR	0.891	0.798	0.776	0.285	0.89
SAW	0.935	0.819	0.801	0.305	0.92

$\rho = 0.95$ has a high value of correlation with TOPSIS, which means a high level of consistency between the proposed method and available MCDM methods, and the DIFSS-WASPAS model also supports temporal dynamics that are not taken into account in the traditional methods which are static.

8.3 *Dynamic vs Static Validation*

Table 12 of the paper will compare the results of the rankings attained through the proposed DIFSS-WASPAS model to the results of the rankings attained through a standard Static WASPAS model that employs the use of single-year data.

Table 12: DIFSS-WASPAS (3-year aggregated) vs Static WASPAS (single-year)

Alternative	DIFSS-WASPAS Score	Rank	Static WASPAS Score	Rank
Hydro	0.952	1st	0.947	1st
Wind	0.828	2nd	0.823	2nd
Solar	0.812	3rd	0.794	3rd
Bioenergy	0.308	4th	0.312	4th

The comparison indicates that both the methods yield primarily similar rankings. Nevertheless, the proposed DIFSS-WASPAS framework includes multi-year data, which is based on the temporal aggregation of data, which enables it to identify the trend of the performance in time. The dynamic capability gives a more realistic view of renewable energy performance as opposed to the single period evaluations.

8.4 Criteria Performance Analysis

Figure 3 depicts the radar chart that indicates the performance of each alternative of renewable energy in the eight evaluation criteria at a normalized level. The figure shows the balanced dominance of hydropower with respect to economic (LCOE, CAPEX), technical (grid stability, capacity factor) and environmental (CO_2 emissions) aspects. The nature of this multidimensional advantage has been the reason why hydropower has always topped the list despite the fact that solar energy has proved to be very good in terms of generation related parameters.

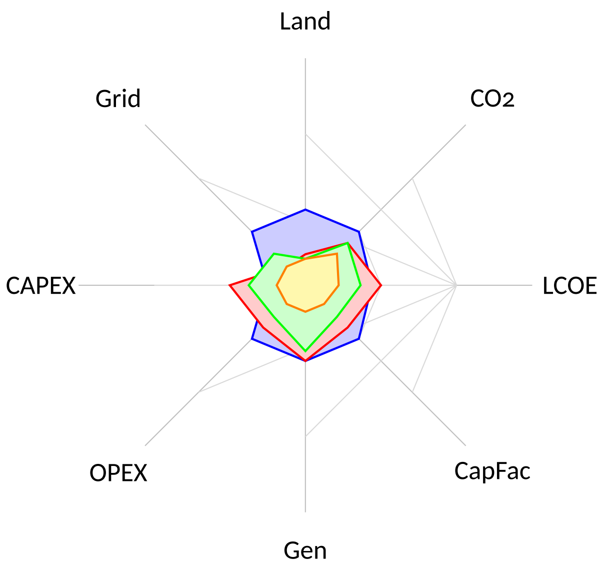

Figure 3: Radar chart illustrating the comparative performance of renewable energy alternatives across eight evaluation criteria. Hydropower demonstrates the most balanced performance, explaining its highest Q_i score.

The results can inform policy-makers in the field of renewable energy planning in Pakistan. Though the case study is based on the Pakistani energy system, the modelling framework proposed can be applied to other countries with few adjustments to the set of alternatives, evaluation criteria and the set of experts.

9. Results and Discussion

The given DIFSS-WASPAS framework was implemented to assess four Pakistani renewable energy options, which included solar, wind, hydropower, and bioenergy. Eight economic, environmental and technical criteria were evaluated including levelised cost of electricity (LCOE), CO2 savings, land use, grid stability, capital expenditure (CAPEX), operational expenditure (OPEX), generation capacity and capacity factor. The last WASPAS aggregation is a combination of the weighted sum model and the weighted product model to get the final performance scores Q_i of each alternative.
The scores obtained show that hydropower has the best overall performance of $Q_i = 0.947$ and then

wind energy with $Q_i = 0.823$, solar energy with $Q_i = 0.794$ and bioenergy with $Q_i = 0.312$. The obtained results give the final ranking: Hydropower $>$ Wind $>$ Solar $>$ Bioenergy.
This performance has been observed as a balanced output of the hydropower in terms of economic, environmental and technical standards. Specifically, hydropower has high ratings in grid stability, generation capacity, and capacity factor, which are very important aspects in ensuring reliable electricity supply in electricity systems with increased renewable penetration. Wind energy is at position two because the cost arrangement is favorable and has a high production capacity whereas the solar energy is at position three regardless of the fast technological advancements. The reason why bioenergy scored lowest is mostly associated with the fact that it is comparatively expensive and has low scaleability in the Pakistani energy scenario.
A key benefit of the suggested framework is that it can bring in information regarding time. As opposed to a one-year snapshot, the DIFSS design combines the expert ratings assessments in several years with recency-based weights ($\lambda_{2021} = 0.24$, $\lambda_{2022} = 0.29$, $\lambda_{2023} = 0.47$). This strategy will be able to seize technological advancement and policy developments and ensure that the decision-making process is not overtaken by temporary fluctuations. As one example, the performance of solar generation stepped up significantly in 2021-2023, but the dynamic aggregation mechanism provides a long-term estimate of the balance instead of exaggerating the latest observations.
The strength of the derived rankings is also proved by a great deal of sensitivity analysis. The ranking of the alternatives does not change when the weights of the dominant criteria are disturbed by $\pm 40\%$ showing that the ranking is perfectly stable. It is also true that when the WASPAS balancing parameter, the value of the parameter λ, and its variation is between 0.2 and 0.8, the numerical values of Q_i change slightly, whereas the relative position of alternatives does not change. These results affirm that the DIFSS-WASPAS model is not sensitive to normal changes in the model parameter and decision-maker preferences.
Other weight vectors to test the scenario analysis were that of cost-oriented and environment-oriented decision-making strategies. In these differing preference structures, the ranking order still stays the same. This shows the proposed framework offers credible decision support within the various priorities of the policy which is significant in the real practice of energy planning as the stakeholder preferences might be quite different.
The effectiveness of the proposed model is further proved by comparative validation against the known multi-criteria methods used to make decisions. The results of the DIFSS-WASPAS show high correlation with other benchmark techniques like TOPSIS, VIKOR and SAW with a Spearman rank correlation of $\rho = 0.95$ with TOPSIS. Nonetheless, in contrast to these traditional methods, the offered framework directly involves the dynamics of time and the element of intuition, which enables it to include changing technological performance and reluctance in judgments of expert The dynamic formulation is also beneficial compared to the traditional WASPAS analysis, which is a static one. By comparing the proposed three-year aggregated dataset with single-year fixed data, more stable and informative assessments are obtained using the dynamic approach. The historical information makes the model more effective in depicting the long-term trends of renewable energy technologies as compared to using only the short-term observations.
At the criteria level, the radar analysis shows that hydropower has high and equal levels of performance in a variety of dimensions, such as economic efficiency, benefits to the environment, and grid reliability. This is the reason it dominates in the final ranking even though it has improved its solar generation capability. The visualization brings out the strengths of various technologies in a complementary manner when compared on the evaluation criteria.

Table 13: Mapping of Study Limitations to Future Directions and Policy Implications

Identified Limitation	Future Research Direction	Policy and Practical Implication
Data intensity: The DIFSS framework depends on multi-period expert inputs, increasing data collection effort.	Employ automated or real-time data acquisition methods; integrate renewable energy databases for continuous model updating.	Enhances evidence-based policy through real-time monitoring and adaptive decision support for dynamic energy systems.
Parameter dependence: The balancing parameter λ and temporal weights affect aggregation outcomes.	Introduce adaptive or entropy-based weighting and optimization schemes for improved parameter calibration.	Enables policymakers to perform sensitivity analyses, improving confidence in energy prioritization and investment decisions.
Geographical scope: Validation limited to Pakistan's renewable energy system.	Apply the DIFSS–WASPAS framework in cross-country studies to generalize findings.	Supports comparative energy policy formulation and fosters regional collaboration in sustainable energy development.
Expert consistency: Assumes stable expert opinions across periods.	Develop dynamic consistency assessment models or machine-learning–based confidence weighting.	Improves reliability of expert-based planning and aligns stakeholder assessments with evolving policy goals.
Scope of application: Current focus restricted to renewable energy planning.	Extend application to smart grids, sustainable urban planning, and real-time resource management.	Facilitates integrated energy transition strategies and supports data-driven infrastructure investment plans.
Technological adoption: Limited focus on hybrid and storage integration.	Explore hybrid renewable systems and storage modeling within dynamic decision environments.	Informs policymakers on optimizing portfolios involving hydropower, wind, and solar technologies for national energy mix alignment.

10. Conclusion

The proposed research presented a dynamic intuitionistic fuzzy soft set WASPAS model in the consideration of renewable energy options in uncertainty and changing time. The model combines intuitionistic fuzzy soft set representation with WASPAS aggregation approach as an aid in the decision-making process of the renewable energy planning. In order to prove the relevance of the proposed approach, a case study of the renewable energy planning in Pakistan was carried out. Eight criteria were applied on four alternatives solar, wind, hydropower, and bioenergy to assess them. The findings show that hydropower has the best overall performance, then there is wind, solar, and bioenergy with the resulting ranking: Hydropower $>$ Wind $>$ Solar $>$ Bioenergy. The analysis also indicates that there are stable rankings in the proposed framework that are generated given various variation of parameters and decision conditions. The contribution of this study is that the time information was factored in the decision-making process. The proposed framework delivers more accurate technological progress and long-term performance trends by integrating multi-year professional analysis using recency-based weights as compared to the traditional fixed methodologies. In turn, the intuitionistic fuzzy soft set WASPAS model is dynamic and enables the assessment of the renewable energy technology in a complex, uncertain environment, which marks it as a reliable decision-support tool. Future studies can build upon this framework by adding other objective weighting methods, implementing the method in other countries or energy systems, and using real-time data to enhance dynamic energy planning and smart grid uses.

Author contributions

Sania Saleem: Formal analysis, Methodology, Writing – original draft. Maria Riaz: Formal analysis, Methodology, Writing – original draft. Fatima Razaq: Data curation, Investigation, Writing – original draft. Muhammad Saeed: Conceptualization, Methodology, Data curation, Investigation, Writing – review & editing, Supervision.

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Conflicts of Interest

The author declares no competing interests.

Data Availability

The authors confirm that the data supporting the findings of this study are available within the article.

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